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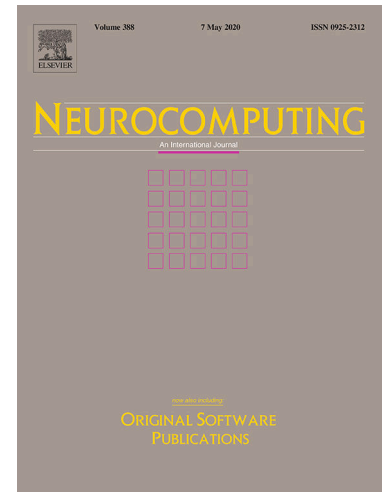
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# An memristor-based synapse implementation using BCM learning rule

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## Abstract

A novel memristive synapse model based on the HP memristor is proposed in this paper, which can address the problem of synaptic weight infinite modulations. The sliding threshold mechanism of the Bienenstock-Cooper-Munro rule (BCM) is used to redefine the memristance (i.e. synaptic weight) adjustment process of the memristive synapse model. Based on the proposed memristor-based synapse and Leaky Integrate-and-Fire neurons, a spiking neural network (SNN) hardware fragment is constructed, where spike trains with different frequencies are used to evaluate the stability performance of the proposed SNN hardware. Results show that compared to other approaches, the network is stable under different stimuli due to the characteristics of the memristor-based synapse model, and prove that the proposed synapse model is able to mimic biological synaptic behaviour and the problem of synaptic weight infinite modulations is addressed.

*Keywords:* memristor, BCM theory, spiking neural networks, learning rule

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## 1. Introduction

Bio-inspired solutions have shown great potential for solving real-world engineering problems, and recent approaches have gained inspirations from biology to improve the reliability of electronic systems [1, 2, 3]. Various approaches

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have been proposed, e.g. using a plastic spiking neural network (SNN) model to develop a fault-resilient robotic controller [4], which has the potential to be applied in robotic obstacle avoidance task. Moreover, such fault-resilient system can maintain stability even under a synaptic fault density of up to 75%. In the approach of [5], a bio-inspired online fault detection and self-correction system for robotic spike-based controller is developed, where the self-detection and self-correction strategy can detect faults and re-allocate resources to restore the controller's functionality. Similarly, an astrocyte-neuron network for self-repairing mobile robotic car is proposed in the approach of [6], which can maintain the system stability through self-repair mechanism of the tripartite synapse. In the approach of [7], the Lyapunov-Krasovskii functional and linear matrix inequality framework are used to maintain the stability of the memristive recurrent neural networks.

In biology, the brain can adapt to external environment changes [8], where the learning mechanism plays a key role and the neural network can remain relatively stable (i.e. homeostatic [9]). Particularly, the Hebbian learning rule is a widely used neural network learning mechanism [10]. However, the conventional Hebbian learning rule has the drawback of unlimited modulation of synaptic weight, which can cause the system to collapse [10, 11]. Therefore, this learning rule was modified by introducing a sliding threshold, i.e. the Bienenstock-Cooper-Munro (BCM) learning rule, which can significantly improve the network stability [11, 12, 13]. Inspired by this, in our previous work [14], a self-repairing learning rule for spiking astrocyte-neuron networks is proposed. It is a combination of the spike-timing-dependent plasticity (STDP) and BCM learning rules, which can guarantee the system performance even with a synaptic fault density approaching 80%. Additionally, memristor is becoming popular in the biological neural networks. Due to that the characteristics of nonvolatility, nonlinearity and scalability, memristor device provides an elegant candidate for building the synapses and hardware neuromorphic systems, and various mathematical models of the memristor devices have been proposed and used for the synapse implementations [15, 16, 17, 18, 19]. However, due to the

physical properties of the memristor, i.e. the synaptic weight changes when it is stimulated under signals from the pre- or postsynaptic neurons, causing the memristor-based synapse faces the prominent problem of weight infinite modulation [11, 20, 21]. Considering that, if the synaptic weight of memristor-based synapse is modulated too high (i.e. memristance is very low), it leads to an over-stimulated network where the postsynaptic neurons collapse [11]. Conversely, if the weight is too low (i.e. memristance is very high), the memristor-based synapse is in an inactive state [11]. Obviously, the synaptic weights affect the stability of the SNNs [22], but the traditional methods cannot well address the problem of synaptic weight infinite modulation. For instance, in order to avoid the synaptic weight modulation problem of the common oxide-based memristor, a binary memristive device of  $\text{HfO}_2$ -based oxide-based resistive memory (OxRAM) with only two distinct resistive states of low and high is applied in [20], where OxRAM cannot completely perform the behaviour of biological synapse in analogue neuromorphic circuit. In addition, some other methods make the weight modulation process of the memristive synaptic devices follow the unsupervised learning rules, e.g. STDP rule, through the specific external modification signal [21, 23]. Generating these external modified signals increases the complexity of the hardware implementation of memristive neuromorphic system. Therefore, different from these approaches, this paper aims to provide a more universal and simpler structure of SNN hardware for the memristive neuromorphic systems, and to solve the problem of synaptic weight infinite modulation of the common oxide-based memristive synapses. In summary, the main contributions of this work are as follows:

(a). A novel memristor-based synapse model is proposed in this paper which mimics the synaptic weight modulation process in the biological neural networks. Different from the conventional memristive synapse model, the proposed model uses the sliding threshold controlling mechanism to redefine the memristance adjustment process, which enables memristive synapses to be regulated by neuronal learning rules.

(b). The problem of synaptic weight infinite modulation in the conventional

memristive synapse is addressed by using the BCM learning rule. Based on the proposed memristor-based synapse model, a SNN hardware fragment is developed to show the scalability, where the change in network synaptic weight depends only on the activity level (i.e. spike rate) of the pre- and postsynaptic neurons.

(c). Results demonstrate that the proposed SNN architecture can maintain stability under the stimuli of continuous input spike trains with different frequencies, which is beneficial to build analogue hardware SNN circuits, and provide an elegant method to explore the reliable neuromorphic systems.

The paper is organized as follows. Section 2 gives the definitions of the learning rule and neuron model. Section 3 presents the memristor-based synapse model and the proposed SNN hardware architecture. Section 4 provides the experimental results under various input stimuli. Section 5 summarizes the paper and discusses the future works.

## 2. Learning Rule and LIF Neuron Model

This section analyses the constraints of Hebbian learning rule and gives a detailed definition of BCM learning rule and the LIF neuron model used in this paper. The Hebbian learning rule is one of the earliest synaptic plasticity learning rules, and it is described by:

$$\tau_w \frac{dw_{ij}}{dt} = v_i u_j, \quad (1)$$

where  $w_{ij}$  is the synaptic weight between neuron  $i$  and  $j$ ,  $v_i$  and  $u_j$  represent the spike rates of the pre- and postsynaptic neurons, respectively.  $\tau_w$  is a time constant that controls the synaptic weight change rate. This conventional Hebbian learning rule is unstable as there is no decay for the synaptic weight to decrease [24]. When both pre- and postsynaptic neurons produce activity simultaneously, the synaptic weight keeps increasing, see the red solid in Fig. 1(a).

The BCM theory introduces the sliding threshold of  $\theta_m$  based on Hebbian learning rule [12]. Its stability has been proved, namely metaplasticity [25, 26].

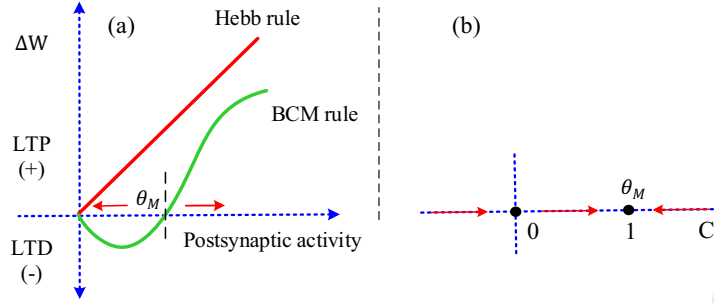


Figure 1: SNN learning rule. (a). BCM and Hebbian learning rules. (b). BCM instantaneous sliding threshold.

According to the BCM theory [27], the synaptic modification  $\dot{m}(t)$  at time  $t$  is given by

$$\dot{m}(t) = \eta \phi(y(t), \theta_m(t)) x(t) - \epsilon m(t), \quad (2)$$

where  $\eta$  is the constant learning rate,  $x$  and  $y$  represent the time-average of pre- and postsynaptic neurons activity, respectively. In this paper, the time-average of neuronal activity over the history is represented by the spike-count rate (i.e. mean frequency), and it can be calculated by  $r = n/T$ , where  $r$  denotes the spike-count rate,  $T$  is the running time and  $n$  is the number of spikes that appear during a trial.  $\theta_m$  is the sliding modification threshold,  $\epsilon m$  is the decay term for synaptic efficacy.  $\phi(\cdot)$  is the nonlinear activation for  $y$  and  $\theta_m$ , and its definition on one-dimensional analysis is given by

$$\phi(y(t), \theta_m(t)) = y(t)(y(t) - \theta_m(t)). \quad (3)$$

In particular, the sliding threshold  $\theta_m$  varies nonlinearly with the continuous time-weighted average output  $\bar{y}(t)$  over the history of the postsynaptic neurons. Hence the  $\theta_m$  is also a frequency, and  $\bar{y}(t)$  is described by

$$\bar{y}(t) = \frac{1}{\tau} \int_{-\infty}^t e^{-(t-t')/\tau} dt', \quad (4)$$

where  $\tau$  is the exponential time window. A more stable form of  $\theta_m$  was proposed later in the approach of [28], and described as

$$\theta_m(t) = \bar{\bar{y}}^p = \frac{1}{\tau} \int_{-\infty}^t y^p(t') e^{-(t-t')/\tau} dt', \quad (5)$$

where the degree of nonlinearity of  $\theta_m$  is set by  $p$ , and  $p = 2$  in this paper [26, 27], and  $\theta_m$  is the squared activity integrated over the exponential time window  $\tau$  which is given by

$$\tau = \int_{-\infty}^t e^{-(t-t')/\tau} dt'. \quad (6)$$

It should be noted that because of the initial dormancy of the synaptic rule links,  $y = 0$  if  $t < 0$  [26]. Then eqn. (5) can be rewritten as

$$\theta_m(t) = \frac{\int_0^t y^2(t') e^{-(t-t')/\tau} dt'}{\int_0^t e^{-(t-t')/\tau} dt'}, \quad (7)$$

where the change speed of  $\theta_m$  affects the activity of postsynaptic neurons. Particularly, if  $\theta_m$  changes too slowly (i.e.  $\tau$  is too large), it will cause large oscillations of postsynaptic neuron activity.  $\theta_m$  will become large with rapid fluctuations if it is updated instantaneously (i.e.  $\tau$  is too small). Obviously, the value of  $\tau$  has a directly effect on the network stability. In order to improve the network stability, the sensitivity analysis of the parameters  $\eta$ ,  $\tau$  and  $x$  is necessary [29, 30]. Thus, according to the previous work of [27], for a given learning rate  $\eta$ , the sensitivity description between these parameters of  $\eta$ ,  $\tau$  and  $x$  can be described as  $\eta\tau x < 1$ , where  $\eta$  is experimentally set to  $\eta = 10^{-5}$  in this paper.

As shown in Fig. 1(a), the sliding threshold  $\theta_m$  oscillates with the level of postsynaptic activity, according to the BCM theory (see the green solid). When the postsynaptic activity level is lower than  $\theta_m$ , a long-term depression (LTD) effect occurs on synaptic weights ( $\Delta W < 0$ ). If it is greater than  $\theta_m$ , a long-term potentiation (LTP) effect happens, i.e.  $\Delta W > 0$ . Due to the sliding threshold and gentle BCM curve, the infinite modulation of the synaptic weight is prevented and the postsynaptic neurons are finally in a dynamic stable state. In combination with the bipolar characteristic of the memristor-based synapse, for the LTD the memristor-based synapse outputs a negative electrical signal, which is contrary to the BCM theory. Therefore, the instantaneous sliding threshold [27] is used in this paper to study the stability of the memristor-based SNNs.



One significant feature of the instantaneous sliding threshold is  $C = mx(t)$ , where  $C$  and  $x$  denote the activity levels of pre- and postsynaptic neurons, respectively,  $m$  denotes the current synaptic weight. Note that  $C$  is different from the time-average activity  $y(t)$  of the postsynaptic neuron in (3). The calculation of  $\theta_m$  is unchanged and the  $y(t)$  in (3) is replaced by  $C$ , so eqn. (3) can be rewritten as

$$\phi(C, \theta_m(t)) = C(C - \theta_m(t)). \quad (8)$$

Fig. 1(b) shows the only stable point in this case is  $\theta_m = C = 1$ . When  $\theta_m = 1$ , the neural network is dynamically stable. This is the key to the instantaneous sliding threshold combined with the memristor-based synapse, and the detailed mathematical analysis will be derived in section 3.

Additionally, the LIF neuron model is used in this work due to its simplistic nature, which can be described by

$$\tau_m \frac{dv}{dt} = -v(t) + R_m \sum_{k=1}^n I_{syn}^k(t), \quad (9)$$

where  $\tau_m$  is the neuronal membrane time constant,  $v$  is the membrane potential,  $R_m$  is the membrane resistance,  $I_{syn}^k(t)$  denotes the current injected by  $k^{th}$  synapse, and  $n$  represents the total number of synapses connected to the neuron. The firing threshold voltage is 9 mv.

### 3. Memristor-based Synapse and MSNN Architecture

In this section, the characteristics of the memristor-based synapse model and the SNN hardware architecture with two neurons are provided. The mathematical definition and characteristic simulations of the memristor are given firstly; then the combination between memristor and BCM rule is analysed. Finally, the proposed two-neuron network architecture is presented in detail.

#### 3.1. Memristor model and its combination with BCM rule

The  $\text{TiO}_2$ -based memristor is used as the synapse model in this paper due to its nonvolatility, nonlinearity and dynamic properties [18]. This type of memristor model was first proposed by HP Labs in 2008 [31], and its corresponding

physical model and symbol are shown in Fig. 2(a) and (b) respectively. It is a two-end component with positive and negative (“+” and “-”), comprising a doped region with  $\text{TiO}_{2-x}$  ( $w$ ) and an undoped region with  $\text{TiO}_2$  ( $D - w$ ), respectively. The doped region has high conductivity, while the undoped region has high insulation, and the total thickness of the two regions is represented by  $D$  as a fixed value of 10 nm. This memristor model is bipolar. If the input is positive current, the thickness of the doped region increases, and the resistance of the memristor decreases. If the input is negative current, the thickness of the undoped region increases and the resistance increases.

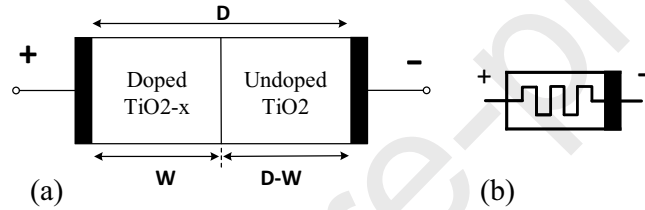


Figure 2: The  $\text{TiO}_2$ -based memristor [31]. (a). Physical model. (b). Memristor symbol.

The  $\text{TiO}_2$ -based memristor model is given by

$$R_{mem}(X(t)) = R_{off} - X(t)\Delta R, \quad (10)$$

where  $R_{mem}$  is the total resistance of the memristor which equals to the sum of the resistance of the doped and undoped regions,  $R_{off}$  and  $R_{on}$  denote the maximum and minimum resistances corresponding to the case of  $w = 0$  and  $w = D$ ,  $\Delta R = R_{off} - R_{on}$ , and  $X(t) = w(t)/D$ .  $X(t)$  represents the ratio between the doped region and the total thickness of the oxide layer. It is always used as the weight of the memristor-based synapse [32], and  $X \in (0, 1)$ . This  $\text{TiO}_2$ -based memristor is a charge-controlled model and its voltage can be described by

$$v(t) = R_{mem}(X(t))i(t), \quad (11)$$

where  $v(t)$  and  $i(t)$  are the voltage and current, respectively, and the change rate of  $X(t)$  is given by

$$dX/dt = ki(t)f(X), \quad (12)$$

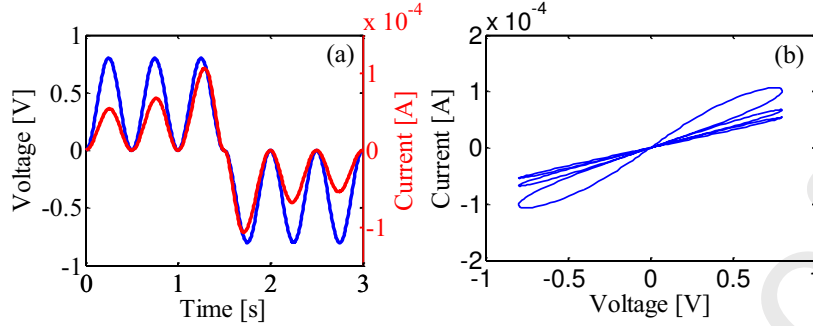


Figure 3: Simulation results of memristor characteristics. The corresponding parameters of the memristor are:  $R_{off} = 20k$ ,  $R_{on} = 100$ ,  $R_{init} = 10k$ ,  $p = 2$ ,  $D = 10nm$ . (a). The memristor current varies with the input voltage. (b). The hysteresis curve of the memristor.

where  $k = \mu_v R_{on}/D^2$ , and  $\mu_v = 10^{-14}m^2s^{-1}v^{-1}$  is the average ion mobility,  $R_{on}$  is the minimum memristor resistance, and  $f(X)$  is the Joglekar's window function [33] which can be described by

$$f(X) = 1 - (2X(t) - 1)^{2p}, \quad (13)$$

where  $p$  controls the degree of nonlinearity, and  $p = 2$  in this paper.

Fig. 3 shows the simulation results of the memristor characteristics with an input of  $v(t) = \pm 0.8 \sin^2(2\pi t)$ . As can be seen by Fig. 3(a), when the input is a positive voltage, the current changes with the voltage and gradually increases, indicating that the memristor resistance is decreasing, and vice versa. Fig. 3(b) shows one of the most significant features of the memristor with this input, i.e. the hysteresis characteristics.

Memristive devices have been used for the SNN synapse implementation based on a specific input signal [11, 23]. However, in addition to synapses, neurons are another key component of biological neural networks. Therefore, a SNN fragment of two neurons and its hardware architecture is proposed in this work, where the BCM is used as the learning rule. Specifically, the instantaneous sliding threshold of the BCM rule is employed to supervise the change process

of the weight of the memristor-based synapse, so that eqn. (12) is rewritten as

$$\frac{dX}{dt} = \begin{cases} ki(t)f(X)\theta(|\dot{\theta}_m|), & \text{if } |\dot{\theta}_m| \neq 0 \\ 0, & \text{if } |\dot{\theta}_m| = 0 \end{cases}, \quad (14)$$

where  $\theta(\cdot)$  is a step function,  $|\dot{\theta}_m|$  is the absolute value of the sliding threshold increment. In particular,  $|\dot{\theta}_m|$  is updated at every iteration, i.e.  $\theta_m$  changes with the activity level of the postsynaptic neuron. Obviously, through eqn. (14), an inherent relationship between the sliding threshold  $\theta_m$ , the weight of memristor-based synapse and the network output is set up.

According to the BCM theory, the sliding threshold changes quickly enough to catch up the neuron activities, and the postsynaptic neurons keep the output by adjusting the synaptic weight [27]. When the output approaches stable state, the instantaneous sliding threshold  $\theta_m$  converges to a constant value (i.e. one), as shown in Fig. 1(b). This property of  $\theta_m$  contributes to the stability of the proposed SNN architecture. As can be seen from eqn. (14), when  $|\dot{\theta}_m| \neq 0$ , the output is not converged. At this time,  $\theta(|\dot{\theta}_m|) = 1$ , then eqn. (14) is equivalent to eqn. (12), i.e. the convergence process of the memristor-based synapse is not affected. When  $|\dot{\theta}_m| = 0$ , the neural network is dynamically stable, and the change rate of the memristor-based synaptic weight is 0, then the output remains constant. Obviously, by combining the BCM sliding threshold with the memristor model, the supervision of the synaptic weight change process is realized.

In addition, the current synaptic efficacy  $m(t)$  in eqn. (2) is considered as the input of the memristor-based synapse at time  $t$ , which is used to analyse the regulation of the BCM mechanism on the memristive synapse. The current  $i(t)$  of (14) can be calculated by

$$i(t) = \frac{m(t)}{R_{off} - X(t)\Delta R}, \quad (15)$$

where  $\Delta R = R_{off} - R_{on}$ . From eqn. (2), (14) and (15), it can be clearly seen that the weight change of the memristor-based synapse depends on the activity levels of the pre- and postsynaptic neurons, which is biologically plausible [34].

At the same time, the change of  $\theta_m$  has a direct impact on the passing current  $i(t)$  of the memristor-based synapse which is used to modulate the synaptic weights (i.e. memristance). By adjusting the synaptic weights, the infinite modulation of the synapse is prevented, and the stability of the network is achieved.

### 3.2. Memristor-based SNN architecture

Fig. 4 illustrates the structure of the proposed SNN fragment. It contains two LIF neurons (i.e. input neuron  $N1$  and output neuron  $N2$ ) and a memristor-based synapse (excitatory). When the presynaptic neuron  $N1$  is excited by the injected current  $I_{in}$ , it outputs a spike train. The rate of the spike train is the input to the synapse, which is denoted by  $x$  in (2). Due to the high initial weight, the output current  $I_m$  of the synapse can induce activity of the postsynaptic neuron  $N2$ . Then the activity level of  $N2$ , i.e. the rate of the output spike train  $y$ , is fed back to the memristor-based synapse, and the change rate of synaptic weight is adjusted by BCM learning rule. Because the sliding modification threshold  $\theta_m$  moves quickly enough to catch up with the activity of the neurons, and it eventually converges to a certain value (i.e. one). At this time, the synaptic weight change rate of memristor-based synapse is 0, as shown in eqn. (14). This causes the output spike frequency of  $N2$  to converge, indicating that the system is dynamically stable. By combining the BCM rule with the memristor model, the problem of synaptic infinite modulation under the continuous stimulus is solved, which provides an alternative method to improve

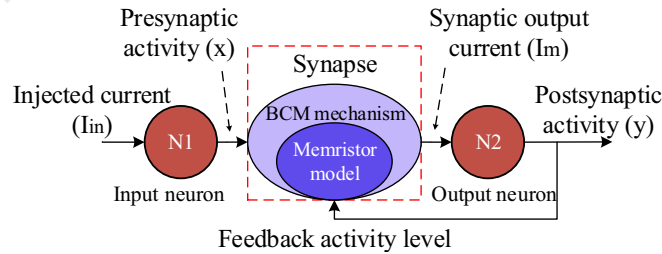


Figure 4: A SNN fragment.

the stability of SNNs. This structure has the potential to be scaled to large networks, i.e. multiple synapses and neurons.

#### 4. Results

This section provides the results of the proposed SNN architecture where the stability is analysed under input spike trains with various frequencies. The Matlab simulation platform is used for all simulations and the memristor modelling. The time step of simulation is set as a fixed value of 1ms which is same as the time step of Euler method of integration. The corresponding parameters of the memristor-based synapse are  $R_{off} = 10k$ ,  $R_{on} = 100$ ,  $R_{init} = 5k$ ,  $p = 2$ ,  $D = 10nm$ , i.e. the initial synaptic weight value is 0.5. According to the BCM instantaneous sliding threshold, the synaptic weight increases in proportion to  $1/x$  [27], where  $x$  is the spike frequency of the presynaptic neuron. A spike train with low frequency is more likely to enhance synaptic efficacy, which is also detailed in [11]. Based on this, the stability of the network is firstly analysed, where the memristor model does not incorporate BCM learning rule. In this case, the memristor-based synaptic weight is changed according to (12), i.e. the BCM rule cannot modulate the synaptic weight change process. The corresponding simulation results are shown in Fig. 5(a)-(d). The spike-count rate is considered as the activity levels of neuron  $N1$ ,  $N2$ , see Fig. 4. When  $N1$  has a continuous stimulus and produces a spike train of  $\sim 10\text{Hz}$ , the sliding threshold  $\theta_m$  can still self-adjust and eventually converge to one, see Fig. 5(a) and (b). However, the synaptic weight is not modulated by the BCM, and it grows indefinitely with the continuous stimulus  $x(t)$ , and eventually exceeds the variation range ( $X \in (0, 1)$ ) of the memristor-based synapse, i.e.  $X = 1$  (Fig. 5(c)). The infinite modulation of the synapse causes the  $N2$  output spike train frequency to increase continuously indicating that the system is unstable, see Fig. 5(d). In order to prevent the infinite modulation of the synapse, the BCM is adopted to modulate the synaptic weight by combining  $\theta_m$  with the memristor model in (14). The experiment is repeated again. As shown by Fig. 5(f), the output

$y(t)$  of  $N2$  tends to stabilize at  $\sim 345\text{Hz}$  as the sliding threshold  $\theta_m$  converges. Meanwhile, compared to the previous result in Fig. 5(c), the synaptic weight is maintained at 0.59, see Fig. 5(e).

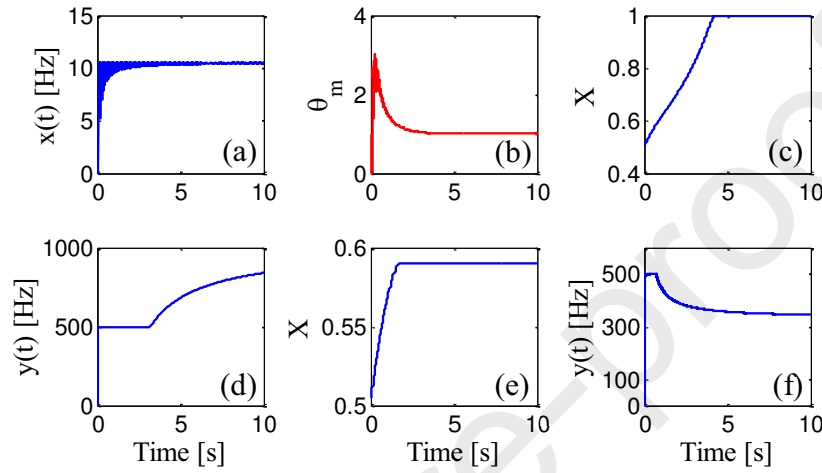


Figure 5: System stability simulation results when the output spike frequency of presynaptic neuron is  $\sim 10\text{Hz}$ .

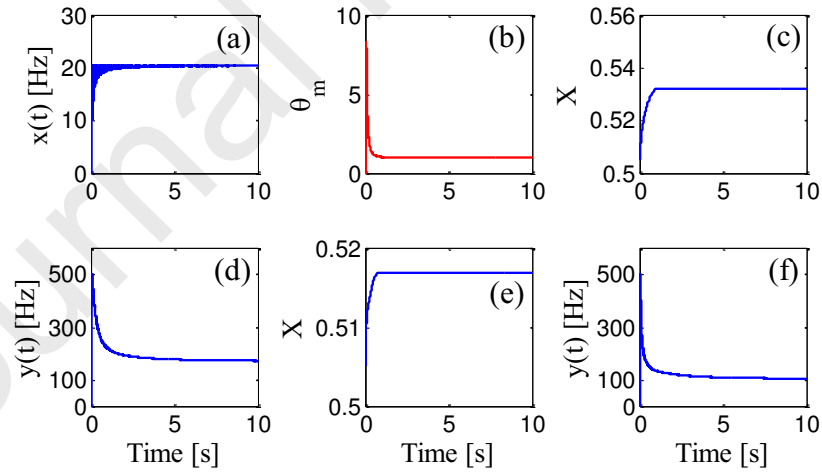


Figure 6: System stability simulation results when the output spike frequency of presynaptic neuron is  $\sim 20\text{Hz}$  (a-d) and  $\sim 30\text{Hz}$  (e, f), respectively.

To further evaluate the stability performance of the proposed architecture, it

is verified by using different input spike trains of  $\sim 20\text{Hz}$  and  $\sim 30\text{Hz}$  respectively. Fig. 6(a)-(d) show the results under the N1 output spike train of  $\sim 20\text{Hz}$ . Similar to the previous discussions, both the synaptic weight  $X$  and the output activity level  $y(t)$  are gradually stabilized at 0.53 and  $\sim 172\text{Hz}$ , respectively, as the sliding threshold  $\theta_m$  converges. A similar profile can also be observed when the input spike train is  $\sim 30\text{Hz}$  as presented in Fig. 6(e) and (f), where the corresponding final weight value  $X$  and  $y(t)$  are 0.52 and  $\sim 104\text{Hz}$ , respectively. Based on the results in Fig. 5 and Fig. 6, it can be seen that the greater the activity intensity of the presynaptic neuron  $N1$ , the smaller the final synaptic weight value. This is consistent with the biological experimental results in the approach of [27]. In addition, the proposed structure contains the neuron and synapse components, which is more complete and adaptable compared to other approaches, e.g. the approaches of [11, 23] are based on one device to mimic biological behaviours of synapse.

In this paper, we aim to address the synaptic weight infinite modulation problem of the common oxide-based analogue memristive synapse, and provide a stable SNN framework for neural network hardware implementations. Table I shows the comparison between different methods for neural network implementations on three different aspects including synapse model, network architecture and learning rule/synthesis methodology (LR/SM). The proposed SNN is constructed by using the analogue memristive synapse model, and can perform the synaptic behaviours of biological synapse, while the approach of [20] uses multiple parallel binary  $\text{HfO}_2$ -based OxRAM cells to model the biological synaptic behaviour at the expense of increased area consumption [21]. In addition, the regulation signal of the proposed memristor-based synapse model is spike trains, while the device programming in the approaches of [21, 23] requires extra circuit elements for monitoring the state of the memristor and shaping the spike accordingly, which consume more area and power. Other approaches of [35, 36, 37] use hybrid algorithm, STEERAGE and SCANN synthesis methodology, to synthesize efficient artificial neural networks for forecasting [35] and pattern recognition [36, 37] tasks. Compared to these approaches [35, 36, 37],



the memristive neuromorphic networks show advantages for hardware implementations [17, 21, 23], especially the high system integration as the memristor is nanodevice with very small area overhead and power consumption [17, 18]. Particularly, due to the fast information processing capability of the hardware circuit system (especially the reaction time of the memristor device can reach the nanosecond level [17]), the memristive SNN hardware circuit has the potential of processing information at the biological time scale.

Table I: Comparison between different methods for neural network implementations

Approaches	Synapse model	Network architecture	LR/SM
[20]	OxRAM	CNN	STDP
[21]	HfO <sub>2</sub> -based	SNN	STDP
[23]	Iron oxide-based	–	STDP & BCM
[35]	–	Hybrid networks	Hybrid algorithm
[36]	–	ANN	SCANN
[37]	–	FFNN & CNN	STEERAGE
This work	TiO <sub>2</sub> -based	SNN	BCM

## 5. Conclusion

An optimized memristor-based synapse model is proposed in this work. By using the proposed synapse and LIF neuron models, a SNN hardware is developed which employs the BCM mechanism as learning rule. The stability of the proposed SNN hardware is evaluated under the input spike trains with different frequencies from 10Hz to 30Hz. Results demonstrate that the SNN has a stable network output and can maintain stability under a continuous excitatory stimulus (represented by spike trains). The proposed memristor-based synapse model is able to mimic biological synaptic behaviour and address the problem of synaptic weight infinite modulations. It has the potential to be scaled to large neural networks due to its compact structure. Future works will explore

the stability of networks on the architecture level and the analogue hardware architecture modelling.

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