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A novel method for interval-value intuitionistic fuzzy multi-criteria decision making problems with immediate probabilities based on OWA distance operators

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Abstract

The goal of this work is to develop a novel decision making method which can solve some complex decision making problems that include the following three aspects information: (1) information represented in the form of interval-valued intuitionistic fuzzy values (IVIFVs) not only intuitionistic fuzzy values, (2) the probability information and the weighted information and (3) the degree of importance of each concept in the process of decision making. Firstly, by integrating OWA operator, probabilistic weight (PW) and individual distance of two IVIFNs in the same formulation, we introduce two new distance operators named PIVIFOWAD operator and IPIVIFOWAD operator, respectively. Secondly, satisfaction degree of an alternative is proposed based on the positive ideal IVIFS and the negative ideal IVIFS and applied to MCDM. Finally, we use an illustrative example to show the feasibility and validity of the new method by comparing with the other existing methods.

Keywords: IVIFS, probability, distance measures, aggregation operators, MCDM

1. Introduction

Atanassov and Gargov [1, 2] introduced the theory of Interval-Valued Intuitionistic Fuzzy Set (IVIFS), which is a generalization of the Intuitionistic Fuzzy Set (IFS) proposed by Atanassov [3], where the membership degree and nonmembership degree of each element belonging to an IVIFS are represented by interval-valued intuitionistic fuzzy values (IVIFVs), which is a subinterval of $[0,1]$, respectively. The IVIFS has received more and more attention since its appearance. Some decision making methods under IVIF environment have been developed by many

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scholars. To sum up, there are mainly four aspects on the decision making under IVIF environment: (1) some decision making methods are developed based on information measures (special-ly, distance, similarity, and entropy) because information measure for IVIFSs have great effects on the development of the IVIFS theory and its applications. Such as similarity measures[5, 20, 31], inclusion measure[45], entropy measure[29], cross-entropy measure[46] and distance measures[42] are developed and applied to corresponding MCDM and MADM problems; (2) many new aggregation operators are also investigated in the IVIFSs and applied to some decision making problems, such as linguistic intuitionistic fuzzy power Bonferroni Mean operators[10], Hamacher aggregation operators[11], fuzzy power Heronian aggregation operators[12], fuzzy generalized aggregation operator[13, 15, 16, 17, 18, 19], (fuzzy Einstein) hybrid weighted aggregation operators[36, 21], fuzzy prioritized hybrid weighted aggregation operator[38], fuzzy Hamacher ordered weighted geometric operator[40] and so on; (3) other methods for decision making with IVIF information are also explored, such as evidential reasoning methodology[4], particle swarm optimization techniques[5], transform technique[?], nonlinear programming methods[39], VIKOR methods in IVIFS[30]and others methods[6, 7, 14, 33, 37, 47] are also developed for decision making problems. Distance measure has great effects on obtaining the desirable choice in some decision problems. Motivated by the OWA operator, Xu [41] introduced ordered weighted distance operator based on known Haming distance. Many extensions of distance operator has been developed, such as Merigo et al. [28] introduced a series of aggregation operator related to distance measures[23, 25, 27, 26] and were applied to related decision problems[24, 27, 34].

In some real decision problems, many problems are very complex. Aim at solving some these complicated decision problems, it is necessary to develop a new kind of decision making method to solve this kind of problems including the following three aspects information: (1) information is represented in the form of IVIFVs not only IFVs, (2) the weighted information and the probability information and (3) the degree of importance of each concept in the process of decision making. Motivated by the ideas of existed operators, we propose new IVIF distance measures by using related weighted operators with probabilistic information, and their applications in MCDM in the present work.

The rest of the paper is organized as follows. In Section 2, we review some related definitions on IVIFSs which are in the analysis throughout this paper. Section 3 is focused on PIVPFOWAD and IPIVIFOWAD. In Section 4, the concept of satisfaction degree is proposed and the MCDM approach based on the satisfaction degree is also constructed. Section 5, a practical example is given to explain proposed method, compare and analyse the validity of proposed MCDM methods. This paper is concluded in Section 6.

2. IVIFSs and OWA Distance Operator

In this section, some related basic concepts of VIFSs, OWA operator and OWAD operator are recapped.

2.1. Interval-valued intuitionistic fuzzy sets

Let $Int([0, 1])$ denote the collection of all closed subintervals of $[0, 1]$, and X be a universe of discourse. An IVIFS $[1, 2]$ on X has such a structure

$$\tilde{A} = \{\langle x, (\mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) \rangle | x \in X\}.$$

where $\mu_{\tilde{A}} : X \rightarrow \text{Int}([0, 1])$ denotes the membership degree and $\nu_{\tilde{A}} : X \rightarrow \text{Int}([0, 1])$ denotes the nonmembership degree of the element $x \in X$ to the set \tilde{A} , respectively, with the condition that $0 \leq \sup(\mu_{\tilde{A}}(x)) + \sup(\nu_{\tilde{A}}(x)) \leq 1$.

For each $x \in X$, $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ denote $\mu_{\tilde{A}}(x) = [\mu_{\tilde{A}}^-(x), \mu_{\tilde{A}}^+(x)]$, $\nu_{\tilde{A}}(x) = [\nu_{\tilde{A}}^-(x), \nu_{\tilde{A}}^+(x)]$, respectively. Therefore, \tilde{A} can also be expressed in another style as follows:

$$\tilde{A} = \{ \langle x, ([\mu_{\tilde{A}}^-(x), \mu_{\tilde{A}}^+(x)], [\nu_{\tilde{A}}^-(x), \nu_{\tilde{A}}^+(x)]) \mid x \in X \}. \quad (1)$$

where Eq.(1) satisfies the condition $\mu_{\tilde{A}}^+(x) + \nu_{\tilde{A}}^+(x) \leq 1$.

$$\pi_{\tilde{A}}(x) = [\pi_{\tilde{A}}^-(x), \pi_{\tilde{A}}^+(x)] = [1 - (\mu_{\tilde{A}}^+(x)) - (\nu_{\tilde{A}}^+(x)), 1 - (\mu_{\tilde{A}}^-(x)) - (\nu_{\tilde{A}}^-(x))]$$

is called the indeterminacy degree. For the convenience, $\tilde{A} = ([\mu_{\tilde{A}}^-, \mu_{\tilde{A}}^+], [\nu_{\tilde{A}}^-, \nu_{\tilde{A}}^+])$ is called an interval-valued intuitionistic fuzzy value (IVIFV).

Hereafter, \mathcal{IVIFV} denotes the collection of all IVIFVs of a IVIFS on X .

For two IVIFVs $\tilde{A}_1 = ([\mu_{\tilde{A}_1}^-, \mu_{\tilde{A}_1}^+], [\nu_{\tilde{A}_1}^-, \nu_{\tilde{A}_1}^+])$ and $\tilde{A}_2 = ([\mu_{\tilde{A}_2}^-, \mu_{\tilde{A}_2}^+], [\nu_{\tilde{A}_2}^-, \nu_{\tilde{A}_2}^+])$, a relation \leq on the IVIFVs is defined as follows:

$$\mu_{\tilde{A}_1}^- \leq \mu_{\tilde{A}_2}^-, \mu_{\tilde{A}_1}^+ \leq \mu_{\tilde{A}_2}^+ \quad \text{and} \quad \nu_{\tilde{A}_1}^- \geq \nu_{\tilde{A}_2}^-, \nu_{\tilde{A}_1}^+ \geq \nu_{\tilde{A}_2}^+. \quad (2)$$

In order to compare two IVIFVs, concepts of score function and accuracy function[32] of an IVIFV are introduced:

For any IVIFV $\tilde{A} = ([\mu_{\tilde{A}}^-, \mu_{\tilde{A}}^+], [\nu_{\tilde{A}}^-, \nu_{\tilde{A}}^+])$, the score function $\tilde{s}(p)$ of \tilde{A} is defined as follows:

$$\tilde{s}(\tilde{A}) = \frac{1}{2}((\mu_{\tilde{A}}^-) + (\mu_{\tilde{A}}^+) - (\nu_{\tilde{A}}^-) - (\nu_{\tilde{A}}^+)), \quad (3)$$

where $\tilde{s}(\tilde{A}) \in [-1, 1]$.

For any IVIFV $\tilde{A} = ([\mu_{\tilde{A}}^-, \mu_{\tilde{A}}^+], [\nu_{\tilde{A}}^-, \nu_{\tilde{A}}^+])$, the accuracy function $\tilde{a}(p)$ of \tilde{A} is defined as follows:

$$\tilde{a}(\tilde{A}) = \frac{1}{2}((\mu_{\tilde{A}}^-) + (\mu_{\tilde{A}}^+) + (\nu_{\tilde{A}}^-) + (\nu_{\tilde{A}}^+)), \quad (4)$$

where $\tilde{a}(\tilde{A}) \in [0, 1]$.

Based on above definitions, comparison rules between two IVIFVs are defined as follows:

- For any two IVIFVs \tilde{A}_1, \tilde{A}_2 ,
- (1) if $\tilde{s}(\tilde{A}_1) < \tilde{s}(\tilde{A}_2)$, then $\tilde{A}_1 < \tilde{A}_2$.
 - (2) if $\tilde{s}(\tilde{A}_1) = \tilde{s}(\tilde{A}_2)$, then
 - (a) if $\tilde{a}(\tilde{A}_1) < \tilde{a}(\tilde{A}_2)$, then $\tilde{A}_1 < \tilde{A}_2$;
 - (b) if $\tilde{a}(\tilde{A}_1) = \tilde{a}(\tilde{A}_2)$, then $\tilde{A}_1 \sim \tilde{A}_2$.

2.2. OWA Distance Operator

In this section, we will review the OWAD operator and then introduce IVIFOWAD operator.

An OWA operator [43] of dimension n is a mapping $OWA : R^n \rightarrow R$ that has an associated weighting $\omega = (\omega_1, \dots, \omega_n)$ with $\omega_j \in [0, 1]$ and $\sum_{j=1}^n \omega_j = 1$, such that

$$OWA(a_1, a_2, \dots, a_n) = \sum_{j=1}^n \omega_j b_j \quad (5)$$

where $(a_1, \dots, a_n) \in R^n$ and b_j is the j th largest of a_i .

70 Let $A = (a_1, \dots, a_n), B = (b_1, \dots, b_n)$ be two collections of arguments. An OWAD operator [25] is a function $OWAD : R^n \times R^n \rightarrow R$ that has an associated weight vector $\omega = (\omega_1, \dots, \omega_n)$ with $\omega_j \in [0, 1]$ and $\sum_{j=1}^n \omega_j = 1$, such that

$$OWAD(A, B) = \sum_{j=1}^n \omega_j d_j \quad (6)$$

where d_j is the j th largest of $|a_i - b_i|$.

Let $\tilde{A} = \{\tilde{\alpha}_i = (\mu_{\tilde{\alpha}_i}, \nu_{\tilde{\alpha}_i})\}$ and $\tilde{B} = \{\tilde{\beta}_i = (\mu_{\tilde{\beta}_i}, \nu_{\tilde{\beta}_i})\}$ be two collections of IVIFVs, where
75 $(\mu_{\tilde{\alpha}_i}, \nu_{\tilde{\alpha}_i}) = ([\mu_{\tilde{\alpha}_i}^-, \mu_{\tilde{\alpha}_i}^+], [\nu_{\tilde{\alpha}_i}^-, \nu_{\tilde{\alpha}_i}^+])$, $(\mu_{\tilde{\beta}_i}, \nu_{\tilde{\beta}_i}) = ([\mu_{\tilde{\beta}_i}^-, \mu_{\tilde{\beta}_i}^+], [\nu_{\tilde{\beta}_i}^-, \nu_{\tilde{\beta}_i}^+])$, $i = 1, 2, \dots, n$. We first recall the distance [42] between two IVIFVs $\tilde{A}_1 = ([\mu_{\tilde{A}_1}^-, \mu_{\tilde{A}_1}^+], [\nu_{\tilde{A}_1}^-, \nu_{\tilde{A}_1}^+])$ and $\tilde{A}_2 = ([\mu_{\tilde{A}_2}^-, \mu_{\tilde{A}_2}^+], [\nu_{\tilde{A}_2}^-, \nu_{\tilde{A}_2}^+])$.

$$\begin{aligned} d(\tilde{A}_1, \tilde{A}_2) &= \frac{1}{4} (|(\mu_{\tilde{A}_1}^-) - (\mu_{\tilde{A}_2}^-)| + |(\mu_{\tilde{A}_1}^+) - (\mu_{\tilde{A}_2}^+)| + |(\nu_{\tilde{A}_1}^-) - (\nu_{\tilde{A}_2}^-)| + |(\nu_{\tilde{A}_1}^+) - (\nu_{\tilde{A}_2}^+)| \\ &\quad + |(\pi_{\tilde{A}_1}^-) - (\pi_{\tilde{A}_2}^-)| + |(\pi_{\tilde{A}_1}^+) - (\pi_{\tilde{A}_2}^+)|). \end{aligned} \quad (7)$$

3. IPIVIFOWA Distance Operator

In this subsection, by combining OWA operator, individual distances and PWs, two new distances named PIVIFOWAD operator and IPIVIFOWAD operator will be introduced. PIVI-
80 FOWAD operator is defined as follows:

Definition 1. A PIVIFOWAD is a function $PIVIFOWAD: I^{\mathcal{V}}IF^{\mathcal{F}}\mathcal{V}^n \times I^{\mathcal{V}}IF^{\mathcal{F}}\mathcal{V}^n \rightarrow \mathbf{R}$, that has an associated weight vector $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ with $\omega_i > 0$, $\sum_{i=1}^n \omega_i = 1$ ($i = 1, 2, \dots, n$), such that

$$PIVIFOWAD(\tilde{A}, \tilde{B}) = \xi \sum_{i=1}^n \omega_i d(\tilde{\alpha}_i, \tilde{\beta}_i) + (1 - \xi) \sum_{j=1}^n p_j d(\tilde{\alpha}_j, \tilde{\beta}_j), \quad (8)$$

where $d(\tilde{\alpha}_i, \tilde{\beta}_i)$ is the i th largest of $d(\tilde{\alpha}_j, \tilde{\beta}_j)$ and each $d(\tilde{\alpha}_j, \tilde{\beta}_j)$ has associate a probabilistic
85 $p_j \in [0, 1]$, $\sum_{i=1}^n p_i = 1$.

In Def.1, if $\xi = 1$, it will be reduced to interval-valued intuitionistic fuzzy ordered weighted distant (IVIFOWAD) operator:

$$IVIFOWAD(\tilde{A}, \tilde{B}) = \sum_{i=j}^n \omega_j d(\tilde{\alpha}_j, \tilde{\beta}_j), \quad (9)$$

where $d(\tilde{\alpha}_j, \tilde{\beta}_j)$ is the j th largest of $d(\tilde{\alpha}_i, \tilde{\beta}_i)$ and $d(\tilde{\alpha}_i, \tilde{\beta}_i)$ is the argument variable represent in the form of individual distance between IVIFVs $\tilde{\alpha}_i, \tilde{\beta}_i$.

90 **Example 1.** Let

$$\begin{aligned} \tilde{A} &= \{([0.3, 0.7], [0.2, 0.3]), ([0.7, 0.8], [0.1, 0.2]), ([0.2, 0.3], [0.5, 0.6]), ([0.5, 0.6], [0.3, 0.4])\}, \\ \tilde{B} &= \{([0.4, 0.5], [0.2, 0.4]), ([0.5, 0.6], [0.2, 0.3]), ([0.2, 0.4], [0.5, 0.6]), ([0.4, 0.7], [0.2, 0.3])\} \end{aligned}$$

be two collections of IVIFVs on the set $X = \{x_1, x_2, x_3, x_4\}$ and the weight vector is $\omega = (0.2, 0.3, 0.1, 0.4)$. Take $\xi = 0.3$, according to Def. 1, we have

$$\begin{aligned} PIVIFOWAD(\tilde{A}, \tilde{B}) &= 0.3 \times (0.2 \times 0.2 + 0.3 \times 0.175 + 0.1 \times 0.175 + 0.4 \times 0.05) \\ &\quad + 0.7 \times (0.3 \times 0.175 + 0.2 \times 0.2 + 0.4 \times 0.05 + 0.1 \times 0.175) = 0.13. \end{aligned}$$

Now, we can also develop the IPIVIFOWAD operator by applying IVIF information, individual distance and immediate probability (IP)[44].

95 **Definition 2.** An IPIVIFOWAD is a function $IPIVIFOWAD : I\mathcal{V}IF\mathcal{V}^n \times I\mathcal{V}IF\mathcal{V}^n \rightarrow \mathbf{R}$ which has an associated weight vector $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ with $\omega_i > 0, \sum_{i=1}^n \omega_i = 1 (i = 1, 2, \dots, n)$, such that

$$IPIVIFOWAD(\tilde{A}, \tilde{B}) = \sum_{i=1}^n \hat{\rho}_i(d(\alpha_i, \beta_i)), \quad (10)$$

where $d(\tilde{\alpha}_i, \tilde{\beta}_i)$ is the i th largest of $d(\tilde{\alpha}_j, \tilde{\beta}_j)$ and $d(\tilde{\alpha}_j, \tilde{\beta}_j)$ is the argument variable represent in the form of individual distance between IVIFVs $\tilde{\alpha}_i, \tilde{\beta}_i$ and a PW $p_i > 0, \sum_{i=1}^n p_i = 1$. $\hat{\rho}_i = \frac{\omega_i p_i}{\sum_{i=1}^n \omega_i p_i}$ and p_i is the probabilistic p_j according to $d(\tilde{\alpha}_i, \tilde{\beta}_i)$, that is, according to the i th largest of the $d(\tilde{\alpha}_i, \tilde{\beta}_i)$.
105

It is worth pointing out that IPIVIFOWAD operator is a good approach for unifying probabilities and IVIFOWAD in some particular situations. But it is not always useful. In order to show why this unification does not seem to be a final model, we could also consider other ways of representing $\hat{\rho}_i$ as in [34].

Example 2. In Exa.1, since the following weight vector $\omega = (0.2, 0.3, 0.1, 0.4)$ and the PW $(0.3, 0.2, 0.4, 0.1)$. Now we aggregate this information according to IPIVIFOWAD. As we have calculated the $d(\alpha_i, \beta_i)$ by employing the Eq.(5) as follows,

$$d(\alpha_1, \beta_1) = 0.175, d(\alpha_2, \beta_2) = 0.2, d(\alpha_3, \beta_3) = 0.05, d(\alpha_4, \beta_4) = 0.175.$$

According to the above distance, we reorder the PW $(0.2, 0.3, 0.1, 0.4)$,

$$\sum_{i=1}^5 \omega_i p_i = (0.2, 0.3, 0.1, 0.4)(0.2, 0.3, 0.1, 0.4)^T = 0.3.$$

110 Therefore, $\hat{\rho}_1 = \frac{\omega_1 p_1}{\sum_{i=1}^5 \omega_i p_i} = \frac{0.2 \times 0.2}{0.3} = 0.133$, Similarly, we have $\hat{\rho}_2 = 0.3, \hat{\rho}_3 = 0.033, \hat{\rho}_4 = 0.534$. Therefore, we have

$$IPIVIFOWAD(\tilde{A}, \tilde{B}) = 0.133 \times 0.2 + 0.3 \times 0.175 + 0.033 \times 0.175 + 0.534 \times 0.05 = 0.112.$$

Monotonicity is a kind of vital property in the research of aggregation operators. The aggregation operator with monotonicity will be more reliable in decision making process. The lack of monotonicity may depress the reliability of the final results. PIVIFOWAD and IPIVIFOWAD are new distance measure, whist are aggregation operators. We can prove PIVIFOWAD and IPIVIFOWAD have the proproperties of boundness, monotonicity and reflexivity. The proof of these properties are similar to Theorem 1-3 in [22].
115

4. Method for MCDM based on IPIVIFOWA operator

4.1. Formal description of MCDM with IVIFs

120 The MCDM with IVIF information can be formally presented as follows:

Let $X = \{x_1, \dots, x_m\}$ be a set of m alternatives, $C = \{C_1, \dots, C_n\}$ the collection of attributes and $\omega = (\omega_1, \dots, \omega_n)^T$ be the weight vector of all attributes, which satisfy $0 \leq \omega_i \leq 1$. Assume that alternative $A_i (i = 1, \dots, m)$ w. r. t. attribute $C_j (j = 1, \dots, n)$ is evaluated by an IVIFVs $C_j(x_i) = ([\mu_{ij}^-, \mu_{ij}^+], [v_{ij}^-, v_{ij}^+]) (j = 1, 2, \dots, n; i = 1, 2, \dots, m)$ and $R_{m \times n} = (C_j(x_i))_{m \times n}$ is an IVIF decision matrix. A new kind MCDM approach will be developed based on the distance operators proposed in the Section 3.

125 For a MCDM problem with IVIFVs, the decision matrix $R = (C_j(x_i))_{m \times n}$ and be constructed or given in advance.

$$R_{m \times n} = \begin{pmatrix} ([\mu_{11}^-, \mu_{11}^+], [v_{11}^-, v_{11}^+]) & ([\mu_{12}^-, \mu_{12}^+], [v_{12}^-, v_{12}^+]) & \cdots & ([\mu_{1n}^-, \mu_{1n}^+], [v_{1n}^-, v_{1n}^+]) \\ ([\mu_{21}^-, \mu_{21}^+], [v_{21}^-, v_{21}^+]) & ([\mu_{22}^-, \mu_{22}^+], [v_{22}^-, v_{22}^+]) & \cdots & ([\mu_{2n}^-, \mu_{2n}^+], [v_{2n}^-, v_{2n}^+]) \\ \vdots & \vdots & \vdots & \vdots \\ ([\mu_{m1}^-, \mu_{m1}^+], [v_{m1}^-, v_{m1}^+]) & ([\mu_{m2}^-, \mu_{m2}^+], [v_{m2}^-, v_{m2}^+]) & \cdots & ([\mu_{mn}^-, \mu_{mn}^+], [v_{mn}^-, v_{mn}^+]) \end{pmatrix}$$

We give the concepts of IVIF-PIS, IVIF-NIS and satisfaction degree before the decision making algorithm is given.

130 Considering that the decision information takes the form of IVIFVs, we utilize the score function Eq.(3) and accuracy function Eq.(4) based comparison approach to identify the IVIF-PIS and the IVIF-NIS. We use A^+ to represent IVIF-PIS and A^- to represent IVIF-NIS, they are determined as following:

$$A^+ = \{ \langle C_j, \max_i s(C_j(x_i)) \mid j = 1, 2, \dots, n; i = 1, 2, \dots, m \rangle, \quad (11)$$

$$A^- = \{ \langle C_j, \min_i s(C_j(x_i)) \mid j = 1, 2, \dots, n; i = 1, 2, \dots, m \rangle. \quad (12)$$

Let D is one of IPIVIFOWAD and PIVIFOWAD, $D(A^+, A_i)$ and $D(A^-, A_i)$ denote the distance of A^+ and alternative A_i and the A^- and alternative A_i , respectively. Motivated by the well-known TOPSIS, we take both $D(A^+, A_i)$ and $D(A^-, A_i)$ into consideration simultaneously rather than separately. This leads naturally to the concept of **satisfaction degree**.

Definition 3. Let $A = \{A_1, \dots, A_m\}$ be a collection of alternatives. The satisfaction degree $\lambda(A_i)$ of a given alternative x_i over the criteria $C_j (j = 1, 2, \dots, n)$ defined as:

$$\lambda(A_i) = \frac{(1 - \varepsilon)[D(A^-, A_i)]}{\varepsilon[D(A^+, A_i)] + (1 - \varepsilon)[D(A^-, A_i)]}, i = 1, 2, \dots, m. \quad (13)$$

140 where ε denotes the **risk preference** of the DM and $\varepsilon \in [0, 1]$: $\varepsilon > 0.5$ means that the decision maker is pessimist; while $\varepsilon < 0.5$ means the opposite. $\varepsilon = 0.5$, satisfaction degree is relative closeness using the classic TOPSIS method. The parameter ε is provided by the decision making in advance. It is obviously that $\lambda(A_i) \in [0, 1] (i = 1, 2, \dots, m)$. The higher the satisfaction degree, the better the alternative.

4.2. Decision algorithm for MCDM with IVIF

- Step 1.** Determine the IVIF-PIS and the IVIF-NIS.
- Step 2.** Calculate the distance between IVIFVs in A and IVIFVs in $A^+(A^-)$ according to Eq.(7);
- Step 3.** Recalculate the probability according to distance calculated in Step 2;
- Step 4.** Compute the distance $D(A^+, A_i)$ of the positive ideal IVIFS A^+ and alternative A_i , the distance $D(A^-, A_i)$ the negative ideal IVIFS A^- and alternative A_i , respectively;
- Step 5.** Calculate the satisfaction degree $\lambda(A_i)$ according to Definition 4. And get the priority of the alternative $A_i (i = 1, \dots, m)$ by ranking $\lambda(A_i) (i = 1, \dots, m)$, the bigger the satisfaction degree $\lambda(A_i)$, the better the alternation A_i .
- Step 6.** End.

5. Case study

In this section, we will give a practical example about the optimal invest strategy to show the application of proposed IPIVIFOWAD and PIVIFOWAD. Let us suppose there is an investment company, which wants to invest a sum of money in the best option (adapted from [8]). There is a panel with five possible alternatives to invest the money:

A1: a car company; A2: a food company; A3: a computer company; A4: an arms company; A5: a TV company.

The investment company must take a decision according to the following five criteria:

- G_1 : the risk analysis;
- G_2 : the growth analysis;
- G_3 : the socialCpolitical impact analysis;
- G_4 : the environmental impact analysis.
- G_5 : Other factors

The five possible alternatives $A_i (i = 1, 2, \dots, 5)$ are to be evaluated using the IVIFVs and construct the IVIF decision matrix as shown in Table 1.

Table 1: IVIF decision matrix

	G_1	G_2	G_3	G_4	G_5
A_1	([0.6,0.7], [0.2,0.3])	([0.6,0.7], [0.1,0.2])	([0.5,0.6], [0.3,0.4])	([0.4,0.5], [0.2,0.3])	([0.5,0.6], [0.3,0.4])
A_2	([0.3,0.4], [0.5,0.6])	([0.2,0.3], [0.6,0.7])	([0.4,0.6], [0.2,0.3])	([0.7,0.7], [0.1,0.2])	([0.4,0.6], [0.1,0.2])
A_3	([0.5,0.6], [0.2,0.4])	([0.5,0.7], [0.2,0.3])	([0.4,0.5], [0.3,0.4])	([0.2,0.3], [0.5,0.6])	([0.3,0.4], [0.4,0.5])
A_4	([0.3,0.4], [0.4,0.5])	([0.5,0.6], [0.3,0.4])	([0.5,0.6], [0.2,0.3])	([0.5,0.6], [0.2,0.3])	([0.5,0.6], [0.2,0.3])
A_5	([0.4,0.6], [0.2,0.4])	([0.5,0.6], [0.2,0.3])	([0.6,0.7], [0.2,0.3])	([0.3,0.4], [0.5,0.6])	([0.6,0.7], [0.2,0.3])

To find the desirable alternative, the expert give the probabilistic weight information as follows: $p = (0.3, 0.3, 0.2, 0.1, 0.1)$. They assume that the importance degree of each characteristics is $w = (0.2, 0.3, 0.1, 0.3, 0.1)$.

5.1. Decision making using IPIVIFOWAD operator

Step 1. Determine the IVIF-PIS A^+ and the IVIF-NIS A^- by the score function and accuracy function which are shown table 2.

Table 2: The results by using score function

	G_1	G_2	G_3	G_4	G_5
s_1	0.4	0.5	0.2	0.2	0.2
s_2	-0.2	-0.4	0.25	0.6	0.35
s_3	0.25	0.35	0.1	-0.3	-0.1
s_4	-0.1	0.2	0.3	0.3	0.3
s_5	0.2	0.3	0.4	-0.2	0.4

we can see from Table 2 that $s_j(G_1)(j = 1, 2, 3, 4, 5)$ all are different, so do $s_j(G_2), s_j(G_3), s_j(G_4), s_j(G_5)(j = 1, 2, 3, 4, 5)$. Therefore, we do not need to compute the accuracy function. And so, IVIF-PIS A^+ and IVIF-NIS A^- are obtained, respectively and shown as follows:

$$\begin{aligned}
 A^+ &= \{ \langle G_1, ([0.6, 0.7], [0.2, 0.3]) \rangle, \langle G_2, ([0.6, 0.7], [0.1, 0.2]) \rangle, \langle G_3, ([0.6, 0.7], [0.2, 0.3]) \rangle, \\
 &\quad \langle G_4, ([0.7, 0.8], [0.1, 0.2]) \rangle, \langle G_5, ([0.6, 0.7], [0.2, 0.3]) \rangle \}, \\
 A^- &= \{ \langle G_1, ([0.3, 0.4], [0.5, 0.6]) \rangle, \langle G_2, ([0.2, 0.3], [0.6, 0.7]) \rangle, \langle G_3, ([0.4, 0.5], [0.3, 0.4]) \rangle, \\
 &\quad \langle G_4, ([0.2, 0.3], [0.5, 0.6]) \rangle, \langle G_5, ([0.3, 0.4], [0.4, 0.5]) \rangle \}.
 \end{aligned}$$

Step 2. Denote $A^+ = \{\tilde{\gamma}_1, \dots, \tilde{\gamma}_5\}, A^- = \{\tilde{\tau}_1, \dots, \tilde{\tau}_5\}, A_i = \{\tilde{\alpha}_{i1}, \dots, \tilde{\alpha}_{i5}\}(i = 1, 2, \dots, 5)$. Now we calculate the distance $d(\tilde{\gamma}_j, \tilde{\alpha}_{ij})(i, j = 1, 2, \dots, 5)$ between IVIFVs $\tilde{\gamma}_j, \tilde{\alpha}_{ij}, d(\tilde{\tau}_j, \tilde{\alpha}_{ij})(i, j = 1, 2, \dots, 5)$ between the IVIFVs $\tilde{\tau}_j, \tilde{\alpha}_{ij}$, respectively. The results can be found in the Table.3 and Table 4.

Table 3: The distance between $\tilde{\alpha}_{ij}$ and $\tilde{\gamma}_j$

	$\tilde{\gamma}_1$	$\tilde{\gamma}_2$	$\tilde{\gamma}_3$	$\tilde{\gamma}_4$	$\tilde{\gamma}_5$
A_1	0	0	0.1	0.3	0.1
A_2	0.3	0.5	0.15	0	0.25
A_3	0.1	0.1	0.2	0.5	0.3
A_4	0.3	0.2	0.1	0.2	0.1
A_5	0.15	0.1	0	0.4	0

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Table 4: The distance between $\tilde{\alpha}_{ij}$ and $\tilde{\tau}_j$

	$\tilde{\tau}_1$	$\tilde{\tau}_2$	$\tilde{\tau}_3$	$\tilde{\tau}_4$	$\tilde{\tau}_5$
A_1	0.3	0.5	0.1	0.3	0.2
A_2	0	0	0.1	0.5	0.3
A_3	0.25	0.4	0	0	0
A_4	0.1	0.3	0.1	0.3	0.2
A_5	0.25	0.4	0.2	0.1	0.3

Step 3. Calculate IP by using the above probabilities and weights according to Table 3 and Table 4. The results are shown in Table 5 and Table 6 .

Table 5: The IP according to Table.3.

	G_1	G_2	G_3	G_4	G_5
IP_1	0.1053	0.1579	0.1053	0.4737	0.1579
IP_2	0.2609	0.3913	0.0435	0.2609	0.0435
IP_3	0.1053	0.1579	0.1053	0.4737	0.1579
IP_4	0.2607	0.3913	0.0435	0.2607	0.0435
IP_5	0.0952	0.4286	0.1429	0.2857	0.0476

Table 6: The IP according to Table.4.

	G_1	G_2	G_3	G_4	G_5
IP_1	0.3158	0.1579	0.0526	0.3158	0.1579
IP_2	0.1053	0.1579	0.1053	0.4737	0.1579
IP_3	0.2857	0.4286	0.0952	0.1429	0.0476
IP_4	0.2857	0.1429	0.0476	0.4286	0.0952
IP_5	0.3158	0.1579	0.1579	0.3158	0.0526

Step 4. Calculate the $IPIVIFOWAD(A_i, A^+)$ and $IPIVIFOWAD(A_i, A^-)$ according to Step 2 and Step 3. For convenience, we denote $IPIVIFOWAD(A_i, A^+)$ and $IPIVIFOWAD(A_i, A^-)$ as $D(A_i, A^+)$ and $D(A_i, A^-)$ ($i = 1, 2, \dots, 5$), respectively. The results are as follows.

$$\begin{aligned}
 D(A_1, A^+) &= 0.0579, D(A_2, A^+) = 0.2979, D(A_3, A^+) = 0.1842, \\
 D(A_4, A^+) &= 0.1957, D(A_5, A^+) = 0.1167. \\
 D(A_1, A^-) &= 0.2474, D(A_2, A^-) = 0.1105, D(A_3, A^-) = 0.2214, \\
 D(A_4, A^-) &= 0.1905, D(A_5, A^-) = 0.2816.
 \end{aligned}$$

Step 5. Calculate the satisfaction degree according to the distance in Step 4. The results can be found in Table 7 under different risk preference ε .

Table 7: Satisfaction degree obtained by IPIVIFOWAD under different risk preference parameter ε

	$\lambda(A_1)$	$\lambda(A_2)$	$\lambda(A_3)$	$\lambda(A_4)$	$\lambda(A_5)$	Ranking
$\varepsilon = 0.1$	0.9747	0.7696	0.9154	0.8976	0.9560	$A_1 > A_5 > A_3 > A_4 > A_2$
$\varepsilon = 0.3$	0.9088	0.4641	0.7372	0.6943	0.8492	$A_1 > A_5 > A_3 > A_4 > A_2$
$\varepsilon = 0.5$	0.8103	0.2707	0.5459	0.4933	0.7070	$A_1 > A_5 > A_3 > A_4 > A_2$
$\varepsilon = 0.7$	0.6468	0.1372	0.3400	0.2944	0.5084	$A_1 > A_5 > A_3 > A_4 > A_2$
$\varepsilon = 0.9$	0.3219	0.0396	0.1178	0.0976	0.2115	$A_1 > A_5 > A_3 > A_4 > A_2$

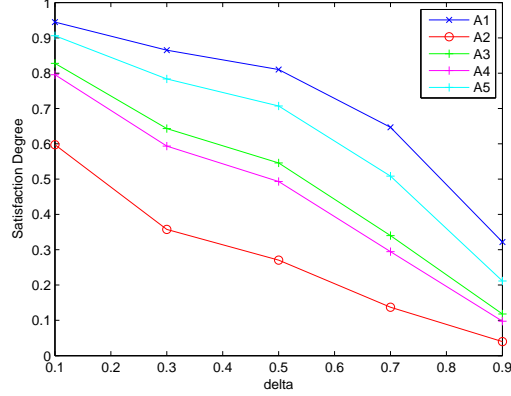


Figure 1: Satisfaction degree obtained by IPIVIFOWAD under different ε

Table 8: The probabilities according to Table.3.

	G_1	G_2	G_3	G_4	G_5
\hat{p}_1	0.14	0.18	0.16	0.3	0.22
\hat{p}_2	0.26	0.3	0.1	0.24	0.1
\hat{p}_3	0.14	0.18	0.16	0.3	0.22
\hat{p}_4	0.26	0.3	0.1	0.24	0.1
\hat{p}_5	0.14	0.3	0.22	0.24	0.1

Table 9: The probabilities according to Table.4.

	G_1	G_2	G_3	G_4	G_5
\hat{p}_1	0.26	0.18	0.1	0.24	0.22
\hat{p}_2	0.14	0.18	0.16	0.3	0.22
\hat{p}_3	0.26	0.3	0.16	0.18	0.1
\hat{p}_4	0.26	0.18	0.1	0.3	0.16
\hat{p}_5	0.26	0.18	0.22	0.24	0.1

It follows from Table 7 that the order of alternatives are consistent with results by using IPIVIFOWAD when parameter changes. We can obtain the ranking $A_1 > A_5 > A_3 > A_4 > A_2$. All of the results show that A_1 is the desirable alternative. Such a conclusion can be drawn directly from Fig.1.

5.2. Decision making using PIVIFOWAD

Now we use the PIVIFOWAD to this decision making problem. According to the distance matrices in Table 3 and Table 4. Note that the WA an importance of 40 percent and the probabilistic information has an importance of 60 percent. we can rearrange the probabilistic according to the distance. The results can be found in the following Table.8 and Table.9.

Therefore, we can calculate PIVIFOWAD distances $D(A_i, A^+), D(A_i, A^-)$ ($i = 1, \dots, 5$) as follows:

$$\begin{aligned}
 D(A_1, A^+) &= 0.076, D(A_2, A^+) = 0.281, D(A_3, A^+) = 0.208, \\
 D(A_4, A^+) &= 0.192, D(A_5, A^+) = 0.123. \\
 D(A_1, A^-) &= 0.228, D(A_2, A^-) = 0.14, D(A_3, A^-) = 0.179, \\
 D(A_4, A^-) &= 0.198, D(A_5, A^-) = 0.271.
 \end{aligned}$$

Therefore, we can obtain the satisfaction degree under difference risk preference parameters ε , please refers the Table.10.

Table 10: Satisfaction degree obtained by PIVIFOWAD under different ε and $\xi = 0.4$

	$\lambda(A_1)$	$\lambda(A_2)$	$\lambda(A_3)$	$\lambda(A_4)$	$\lambda(A_5)$	Ranking
$\varepsilon = 0.1$	0.9643	0.8177	0.8857	0.9027	0.9520	$A_1 > A_5 > A_4 > A_3 > A_2$
$\varepsilon = 0.3$	0.8750	0.5376	0.6676	0.7064	0.8372	$A_1 > A_5 > A_4 > A_3 > A_2$
$\varepsilon = 0.5$	0.7500	0.3325	0.4625	0.5077	0.6878	$A_1 > A_5 > A_4 > A_3 > A_2$
$\varepsilon = 0.7$	0.5625	0.1760	0.2694	0.3065	0.4857	$A_1 > A_5 > A_4 > A_3 > A_2$
$\varepsilon = 0.9$	0.2500	0.0525	0.0873	0.1028	0.1967	$A_1 > A_5 > A_4 > A_3 > A_2$

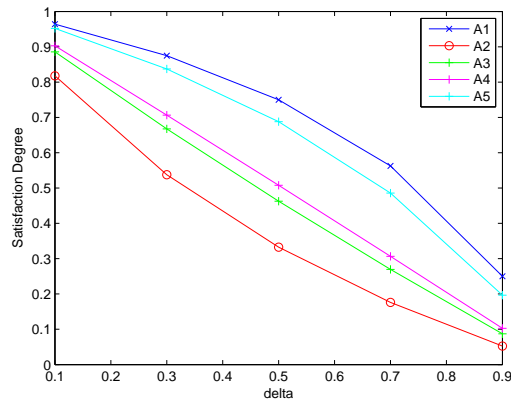


Figure 2: Satisfaction degree obtained by PIVIFOWAD under different ε and $\xi = 0.4$

It follows from Table 10 that the ranking are consistent with the results by using the PIVIFOWAD operators when parameter changes. We can obtain the order of alternatives: $A_1 > A_3 > A_5 > A_4 > A_2$. All of the results show that A_1 is the desirable one. Such a conclusion can be also drawn directly from the Fig.2.

205 If we change the weight important degree ξ , we can obtain other satisfaction degree listed in Table 11. From Table 11, we can see that the desirable alternative is consistent with the ranking

Table 11: Satisfaction degree obtained by PIVIFOWAD under difference ξ

	$\lambda(A_1)$	$\lambda(A_2)$	$\lambda(A_3)$	$\lambda(A_4)$	$\lambda(A_5)$	Ranking
$\xi = 0.1$	0.7762	0.2740	0.5053	0.4923	0.7008	$A_1 > A_5 > A_3 > A_4 > A_2$
$\xi = 0.3$	0.7584	0.3136	0.4766	0.5027	0.6921	$A_1 > A_5 > A_4 > A_3 > A_2$
$\xi = 0.5$	0.7419	0.3509	0.4487	0.5128	0.6835	$A_1 > A_5 > A_4 > A_2 > A_3$
$\xi = 0.7$	0.7267	0.3859	0.4217	0.5231	0.6751	$A_1 > A_5 > A_4 > A_3 > A_2$
$\xi = 0.9$	0.7126	0.4190	0.3955	0.5333	0.6667	$A_1 > A_5 > A_4 > A_2 > A_3$

obtained by PIVIFOWAD when parameter ξ changes although the ranking of alternatives is not

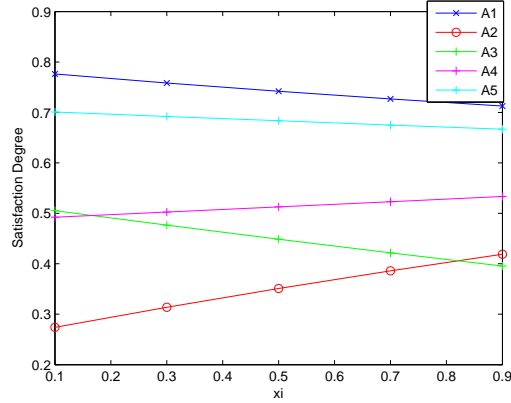


Figure 3: Satisfaction degree obtained by PIVIFOWAD under different ξ and $\varepsilon = 0.5$

the same. All of the results show that A_1 is the desirable one. Such a conclusion can be drawn directly from the Fig.3.

210 *5.3. Effectiveness test of the proposed method*

For MCDM problems, Wang and Triantaphyllou [35] established assessing criteria (please refer to [35]) to assess the effectiveness of MCDM methods. In what following, we will use above MCDM criteria to test our proposed methods in Section 4. As far as the proposed method based on IPIVIFOWAD is concerned, we choose the satisfactory degree $\varepsilon = 0.5$ to analyze above

215 criteria.

Validity test for criterion 1. In Section 5.2, we obtained A_1 is the desirable one and the order of alternatives is: $A_1 > A_5 > A_3 > A_4 > A_2$. In order to test the effectiveness of the developed IPIVIFOWAD method under criterion 1, the modified IVIF decision matrix (Table.12) is used. This decision matrix is gotten by interchanging the intervals of membership and non-membership grades of alternative A_3 (non-optimal) and A_4 (less desirable than A_3) in the original decision matrix (Table 1).

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Table 12: Modified IVIF decision matrix

	G_1	G_2	G_3	G_4	G_5
A_1	([0.6,0.7], [0.2,0.3])	([0.6,0.7], [0.1,0.2])	([0.5,0.6], [0.3,0.4])	([0.4,0.5], [0.2,0.3])	([0.5,0.6], [0.3,0.4])
A_2	([0.3,0.4], [0.5,0.6])	([0.2,0.3], [0.6,0.7])	([0.4,0.6], [0.2,0.3])	([0.7,0.7], [0.1,0.2])	([0.4,0.6], [0.1,0.2])
A_3	([0.2,0.4], [0.5,0.6])	([0.2,0.3], [0.5,0.7])	([0.3,0.4], [0.4,0.5])	([0.5,0.6], [0.2,0.3])	([0.4,0.5], [0.3,0.4])
A_4	([0.4,0.5], [0.3,0.4])	([0.3,0.4], [0.5,0.6])	([0.2,0.3], [0.5,0.6])	([0.2,0.3], [0.5,0.6])	([0.2,0.3], [0.5,0.6])
A_5	([0.4,0.6], [0.2,0.4])	([0.5,0.6], [0.2,0.3])	([0.6,0.7], [0.2,0.3])	([0.3,0.4], [0.5,0.6])	([0.6,0.7], [0.2,0.3])

Repeating the same steps 1-2 in 5.1. We can obtain the modified IVIF-PIS A^+ and the IVIF-

NIS A^- and is listed as follows:

$$\begin{aligned}
 A^+ &= \{ \langle G_1, ([0.7, 0.8], [0.2, 0.3]) \rangle, \langle G_2, ([0.7, 0.8], [0.2, 0.3]) \rangle, \langle G_3, ([0.8, 0.9], [0.3, 0.4]) \rangle, \\
 &\quad \langle G_4, ([0.7, 0.8], [0.3, 0.5]) \rangle, \langle G_5, ([0.7, 0.8], [0.4, 0.5]) \rangle \}, \\
 A^- &= \{ \langle G_1, ([0.2, 0.4], [0.7, 0.8]) \rangle, \langle G_2, ([0.3, 0.4], [0.6, 0.7]) \rangle, \langle G_3, ([0.3, 0.4], [0.6, 0.8]) \rangle, \\
 &\quad \langle G_4, ([0.3, 0.5], [0.5, 0.6]) \rangle, \langle G_5, ([0.3, 0.4], [0.5, 0.7]) \rangle \}.
 \end{aligned}$$

Using the Step 3-Step 5 of IPIVIFOWAD method, the IPIVIFOWAD distances $D(A_i, A^+)$ between alternatives A_i and A^+ , the IPIVIFOWAD distances $D(A_i, A^-)$ between alternatives A_i and A^- are calculated, respectively, where $i = 1, 2, \dots, 5$.

$$\begin{aligned}
 D(A_1, A^+) &= 0.0579, D(A_2, A^+) = 0.2978, D(A_3, A^+) = 0.3452, \\
 D(A_4, A^+) &= 0.3789, D(A_5, A^+) = 0.1167. \\
 D(A_1, A^-) &= 0.3786, D(A_2, A^-) = 0.3526, D(A_3, A^-) = 0.0974, \\
 D(A_4, A^-) &= 0.1, D(A_5, A^-) = 0.3243.
 \end{aligned}$$

According to the satisfaction degree formula when $\varepsilon = 0.5$, we have

$$\lambda(A_1) = 0.8674, \lambda(A_2) = 0.5421, \lambda(A_3) = 0.2200, \lambda(A_4) = 0.2088, \lambda(A_5) = 0.7355.$$

We can see from above satisfaction degrees that the rank is $A_1 > A_5 > A_2 > A_3 > A_4$, that is, A_1 is the best one. Therefore, the best alternative coincide with the best alternative obtained in Section 5.1 by the same method, and the relative orders of the rest of the unchanged alternatives keep constant. That is, Criterion 1 is suitable for the proposed method.

Validity test for criterion 2 and criterion 3. According to the requirements of criterion 2 and test criterion 3 introduced in [35], the original problem should be decomposed into two smaller MCDM problems, such as $\{A_1, A_2, A_3, A_4\}$ and $\{A_1, A_3, A_4, A_5\}$. For the sub-problem $\{A_1, A_2, A_3, A_4\}$, we can obtain the satisfaction degree by repeating Step 1 to Step 6 as follows:

$$\lambda(A_1) = 0.7988, \lambda(A_2) = 0.2859, \lambda(A_3) = 0.56048, \lambda(A_4) = 0.48488.$$

Therefore, the rankings of the sub-problem is $A_1 > A_3 > A_4 > A_2$. For the sub-problem $\{A_1, A_3, A_4, A_5\}$, we can obtain the satisfaction degree by repeating Step 1 to Step 6 as follows:

$$\lambda(A_1) = 0.8577, \lambda(A_3) = 0.38, \lambda(A_4) = 0.2740, \lambda(A_5) = 0.6066.$$

the rankings of the sub-problem $\{A_1, A_3, A_4, A_5\}$ is $A_1 > A_5 > A_3 > A_4$.

We obtain the final ranking $A_1 > A_5 > A_3 > A_4 > A_2$ by combining the order of alternatives of sub-problems $\{A_1, A_2, A_3, A_4\}$ and $\{A_1, A_3, A_4, A_5\}$, the final order is the same with the order of original decision problem and it also exhibits transitive property. Criterion 2 and criterion 3 proposed in [35] also suitable for the proposed method.

5.4. Comparison with existing work

Comparison with Hadi-Vencheh and Mirjaberi's method[9]. In the classical TOPSIS method, we often need to compute the relative closeness of the alternative A_i w.r.t the PIS A^+ as below:

$$RC(A_i) = \frac{D(A_i, A^-)}{D(A_i, A^-) + D(A_i, A^+)} \quad (14)$$

where $D(\cdot)$ is a distance measure. The ranking of all alternatives can be determined according to the closeness index $RC(A_i)$. If $\varepsilon = 0.5$ in our proposed Eq.(15), then Eq.(15) will be Eq.(16).

However, Hadi-Vencheh and Mirjaberi[9] suggested that one may use the following formula instead of the relative closeness index

$$\zeta(A_i) = \frac{D(A_i, A^-)}{D_{max}(A_i, A^-)} - \frac{D(A_i, A^+)}{D_{min}(A_i, A^+)} \quad (15)$$

where $D_{max}(A_i, A^-) = \max_{1 \leq i \leq m} \{D(A_i, A^-)\}$ and $D_{min}(A_i, A^+) = \min_{1 \leq i \leq m} \{D(A_i, A^+)\}$. Eq.(15) is called the revised closeness used to measure the extent to which the alternative A_i is close to the PIS A^+ and is far away from the NIS A^- , simultaneously. By Eq. (15)

$$\zeta(A_1) = -0.1213, \zeta(A_2) = -4.751, \zeta(A_3) = -2.3954, \zeta(A_4) = -2.7029, \zeta(A_5) = -1.0151$$

Therefore, the ranking of $\{A_1, A_2, A_3, A_4, A_5\}$ is arranged $A_1 > A_5 > A_3 > A_4 > A_2$. which coincide with our proposed method.

6. Conclusion

IVIFSs, which is a generalization of the IFSs, have been used widely in decision problems. IVIFS permits the membership degrees and nonmembership degrees to a given set to have an interval value in $[0, 1]$, can be considered as a powerful tool to express complex information in the human decision making process. In this paper, we introduced some new distance measures, namely PIVIFOWAD operator and IPIVIFOWAD operator. Whilst, with respect to probabilistic decision-making problems with IVIF information, some new probabilistic decision-making analysis methods are developed. The new distance operators such as IVIFOWAD operator, PIVIFOWAD operator and IPIVIFOWAD operator have been developed in this paper. Whilst, the concept of satisfaction degree of alternatives has been introduced based on the some distance measures and applied to MCDM problem with IVIF information.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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