



Optimizing MIMO Detection with DM-Detnet in 6G Networks

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Optimizing MIMO Detection with DM-Detnet in 6G Networks

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Abstract—Artificial intelligence (AI) has transformed multiple inputs multiple output (MIMO) technology into a key enabling technology for beyond fifth-generation (B5G) technology such as sixth-generation (6G) networks. However, MIMO technology faces some crucial research challenges, among which signal detection is a significant problem. This paper presents an optimized deep learning-based MIMO detection method called deep MIMO detection network (DM-Detnet) for MIMO detection. The light network architecture of DM-Detnet allows signal detection to be performed in a layer-by-layer manner. This work primarily concentrates on the effect of signal-to-noise ratio (SNR) points on network training and testing. We conducted a detailed simulation study to analyze the performance of our model based on specific low and high SNR points. With intensive training and optimization, the DM-Detnet model achieves better performance in MIMO scenarios. Simulation results show that the optimized DM-Detnet achieves better symbol error rate (SER) performance and is also able to achieve lower computational complexity than benchmark conventional MIMO detectors.

Index Terms—Deep Learning, 5G and Beyond, 6G, MIMO detection, Complexity

I. INTRODUCTION

Every year, there is a remarkable increase in both the quantity of mobile devices and mobile data traffic. The global mobile user count was 7.1 billion in 2021 and reached 7.26 billion by 2022, this figure is expected to hit 7.49 billion by 2025 [1]. To fulfill the ever-increasing demand, it is necessary to have improved spectral efficiency, faster data rates, and greater network capacity. To meet user needs, the Fifth Generation (5G) networks, have already been deployed in the world's biggest cities [2]. Now research era is interested more in a discussion of Sixth Generation (6G) networks. 5G networks are composed of multiple technologies in which Massive Multiput Input Multiple Output (Ma-MIMO) plays a promising candidate role in building the B5G networks [3]. Ma-MIMO is the most captivating wireless access technology to meet B5G network requirements. Hundreds or even thousands of antennas connected to a base station are used in Ma-MIMO technology to increase throughput and spectrum efficiency [4]. However, due to the hundreds of antennas, accurate signal detection became a very challenging issue with a huge cost and complexity [2]. Building an efficient MIMO detector at the receiver end is challenging.

Researchers have developed multiple conventional and AI-based MIMO detectors to address the signal detection problem in MIMO and Ma-MIMO technology. Although linear detectors such as minimum mean square error (MMSE) [5] and zero-forcing (ZF) [6] in MIMO and Ma-MIMO systems provide a better trade-off between Symbol error rate (SER) and computational complexity. However, these linear detectors involve complex matrix inversion. Therefore, due to high-dimensional complex matrix inversion and operations, ZF and MMSE algorithms are difficult to implement in practice [3]. As an alternative to ZF and MMSE detectors, the Maximum-likelihood detector (MLD) is a standard MIMO detector that operates in a nonlinear manner and may theoretically attain the best error performance. However, as the number of antennas increases will increase system complexity [7]. Therefore, in MIMO systems, finding a balance between computational complexity and detection performance has been a key concern. In recent years, machine learning algorithms have been used in several industries, including self-driving automobiles, energy, healthcare, and transportation [8]. To enhance system performance in terms of latency, security, and spectrum usage, these algorithms are also utilized in communication technologies. As machine learning techniques—particularly deep learning techniques are evolving more quickly than ever, it is important to consider SER and complexity by considering these AI-based algorithms [9]. Fig. 1 depicts the integration of AI, which replaces the majority of the processing blocks with machine learning (ML) and it eventually forms an end-to-end learning between the transmitter and receiver which is a broader vision of AI for B5G and 6G network [10]. Based on this motivation, this research paper proposes an optimized AI-based signal detection technique based on DL called DM-Detnet. The proposed DM-Detnet model is an extension of our previous work [11, 12]. The DM-Detnet model enables layer-by-layer signal detection. Each layer's design is based on sophisticated signal detection techniques, leading to their differing performance and complexity against the benchmark conventional detectors for MIMO systems. More specifically, in this work, we concentrate on the effect of SNR points on network training and testing. We optimized our previous work [11, 12] and conducted a simulation study to analyze our model performance based on specific low and high SNR

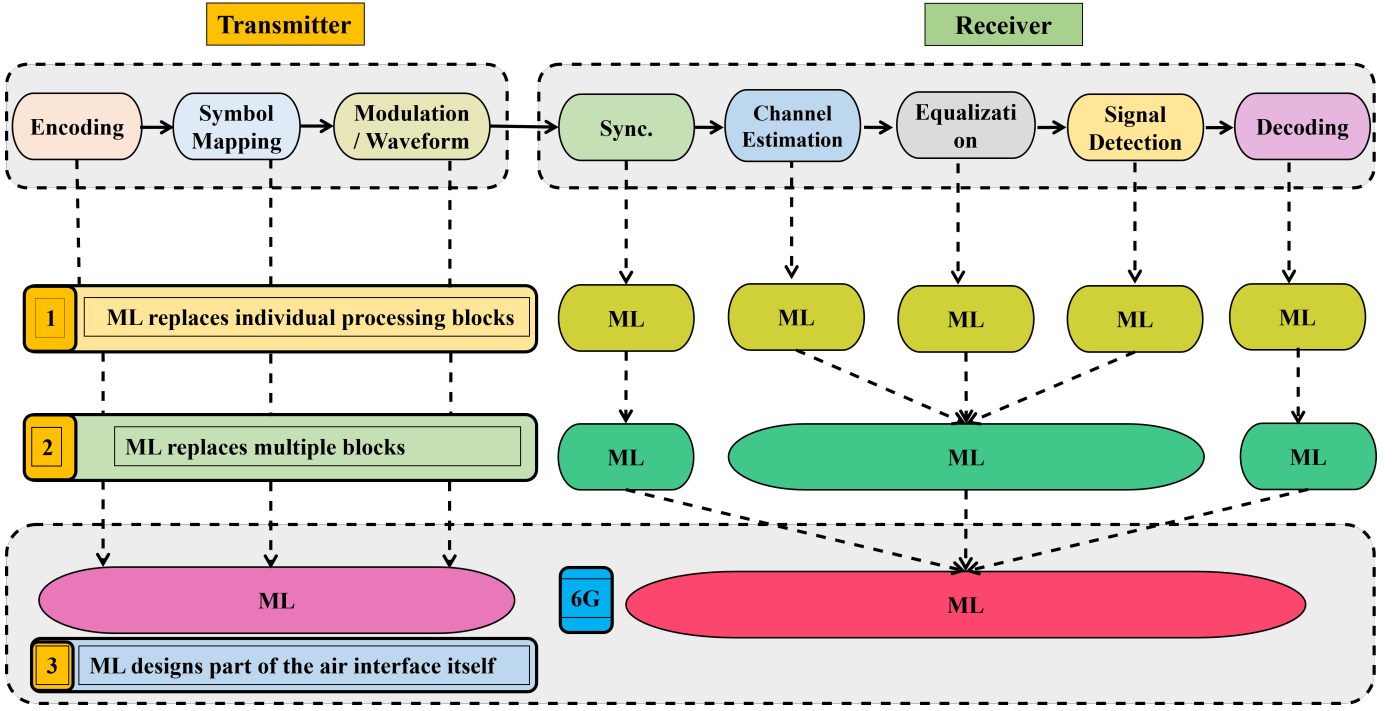


Fig. 1: The integration of AI to form an End-to-End learning for B5G and 6G networks.

points. The primary contribution of this paper is given as follows:

- This paper proposes an AI-based signal detection method called DM-Detnet for MIMO detection.
- Optimized and analyzed our model DM-Detnet by conducting a study on specific SNR points.
- Compute and analyze the DM-Detnet based on SER and complexity against conventional MIMO detectors.

The rest of the paper is organized as follows: The MIMO system model and conventional MIMO detectors are presented in Section II. The proposed optimized DL-based detection method is covered in Section III. The simulation setup and results are presented in Section IV. Finally, Section V addresses the paper's conclusion and future direction.

II. MIMO DETECTION: A SYSTEM MODEL

A MIMO system model consists of N transmitting antennas at the transmitter and M receiving antennas at the receiver respectively. The transmitted symbol vector can be represented by a $N \times 1$ vector $\tilde{\mathbf{t}} = [\tilde{t}_1, \tilde{t}_2, \tilde{t}_3, \dots, \tilde{t}_N]$ with each component $\tilde{t}_i \in \tilde{\theta}$. Consider i.i.d Rayleigh-fading channels denoted by matrix $\tilde{\mathbf{H}} \in \mathbb{C}^{NM}$ that follows independent gaussian distribution, the channel output signal $\tilde{\mathbf{r}}$ is given by equation 1:

$$\tilde{\mathbf{r}} = \tilde{\mathbf{H}}\tilde{\mathbf{t}} + \tilde{\mathbf{n}} \quad (1)$$

where $\tilde{\mathbf{n}}$ denotes the additive white gaussian noise (AWGN) such that $\tilde{\mathbf{n}} \sim \mathcal{CN}(0, \tilde{\sigma}_n^2 I_N)$. Now the complex-valued system in Eq. (1) can be reformulated into an equivalent real-valued system by real value decomposition. The following equations

show how to convert the complex-valued model to a real-valued one,

$$\begin{bmatrix} \text{Re}\{\tilde{\mathbf{r}}\} \\ \text{Im}\{\tilde{\mathbf{r}}\} \end{bmatrix} = \begin{bmatrix} \text{Re}\{\tilde{\mathbf{H}}\} - \text{Im}\{\tilde{\mathbf{H}}\} \\ \text{Im}\{\tilde{\mathbf{H}}\} \quad \text{Re}\{\tilde{\mathbf{H}}\} \end{bmatrix} \begin{bmatrix} \text{Re}\{\tilde{\mathbf{t}}\} \\ \text{Im}\{\tilde{\mathbf{t}}\} \end{bmatrix} + \begin{bmatrix} \text{Re}\{\tilde{\mathbf{n}}\} \\ \text{Im}\{\tilde{\mathbf{n}}\} \end{bmatrix} \quad (2)$$

In this way, the MIMO channel model can be written as:

$$\mathbf{r} = \mathbf{H}\mathbf{t} + \mathbf{n} \quad (3)$$

The mathematical architecture of the MIMO system model is presented in Fig. 2, as follows:

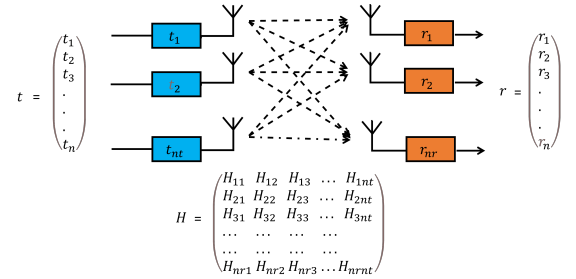


Fig. 2: MIMO system model architecture.

A. Conventional MIMO Detection

Conventional MIMO detectors have made a substantial contribution to our understanding of MIMO systems despite these drawbacks, opening the door for more advanced methods. However, due to intrinsic channel estimate flaws and feedback constraints, conventional MIMO detectors assume that accurate CSI is available at the receiver, which might not be necessary [13]. This section discusses some of the following most commonly used conventional MIMO detectors.

1) *Maximum likelihood detection*: Maximum Likelihood Detection (MLD) is an ideal detector for SER that reduces the risk of mistakes [14, 15]. The following Equation 4 represents the ML detector's mathematical form:

$$\hat{\mathbf{t}}_{\text{MLD}} = \arg \min_{\mathbf{t} \in \mathcal{A}^N} \|\mathbf{r} - \mathbf{H}\mathbf{t}\|^2 \quad (4)$$

where $\hat{\mathbf{t}}$ is the vector approximation of the transmitted signal \mathbf{t} and \mathcal{A}^N shows the finite set of possible symbols in a QAM constellation. However, MLD presents a computational complexity issue, particularly when the number of transmitted and received antennas grows [14, 16].

2) *Zero Forcing*: The Zero Forcing (ZF) method performs well at high SNR points since it makes it possible to assume that the noise is considerably less than the signal being received [16]. The mathematical functionality of the ZF detector is given in Equation 5:

$$\hat{\mathbf{t}}_{\text{ZF}} = \arg \min_{\mathbf{a} \in \mathcal{A}} |z - a| \quad (5)$$

where z is $H^T \mathbf{r} = (H^T H)^{-1} H^T \mathbf{r}$. However, when employing the ZF signal detection approach with ill-conditioned channel matrices, the noise level may rise [14].

3) *Minimum Mean Square Error*: The Minimum Mean Square Error (MMSE) method is introduced to prevent noise amplification in ZF. The mathematical framework for MMSE is represented by Equation 6:

$$\hat{\mathbf{t}}_{\text{MMSE}} = \arg \min_{\mathbf{a} \in \mathcal{A}} |E(t|r) - a| \quad (6)$$

where E is the mean symbol energy. However, for large antenna systems, the MMSE signal detection approach may not be computationally efficient [14, 16].

III. DM-DETNET: A PROPOSED MIMO DETECTOR

The deep neural networking (DNN) model serves as the foundation for the development of the proposed DM-Detnet for MIMO systems. We generate data for training and testing of the DM-Detnet based on transmitted signal \mathbf{t} , channel matrix \mathbf{H} , and AWGN noise \mathbf{n} . After obtaining the received signal \mathbf{r} , at the receiver end, the DM-Detnet scheme is integrated with the received signal \mathbf{r} and channel information \mathbf{H} along with the additive Gaussian noise vector \mathbf{n} as inputs at the receiver end, to recover the transmitted signal \mathbf{t} in the form of $\hat{\mathbf{t}}$. Fig. 3, shows a systematic architecture of the DM-Detnet model.

A. DM-Detnet Network Architecture

The DM-Detnet network model is based on a DNN model-type structure which only has five layers including one input layer, one output layer, and three hidden layers. The DM-Detnet network model has two components that collectively make up the input layer, the received signal \mathbf{r} and the channel matrix \mathbf{H} . The real and imaginary parts of the channel matrix \mathbf{H} and the received signal \mathbf{r} are fed to the first input layer of the DM-Detnet network model. After receiving the input data from the input layer the DM-Detnet network model is further broken down into three hidden layers, with a different number

of neurons in each single hidden layer. The first hidden layer, which is the input combination of the real and imaginary parts from the input layer depicted by the yellow color consists of 64 neurons. The second hidden layer is shown by the light green color which receives an input in-from of the first hidden layer output. The second hidden layer consists of 32 neurons. Similar to the first and second hidden layers the third hidden layer is shown by the light blue color consisting of 16 neurons respectively. The output of one hidden layer becomes the input of the next hidden layers until it reaches the final layer called the output layer consisting of the $\hat{\mathbf{t}}$ part. The network architecture is presented in Fig. 3.

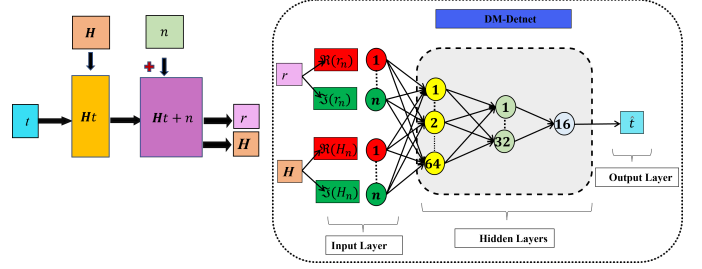


Fig. 3: Systematic architecture of the DM-Detnet model.

B. DM-Detnet: Training and Testing

To train and test the DM-Detnet model, we generate the data based on Eq. 3 with the channel matrix \mathbf{H} , transmitter \mathbf{t} , and noise \mathbf{n} . The training data is produced by the receiver \mathbf{r} calling a functional block that replicates the MIMO transmitter \mathbf{t} activity. In the first step, after initializing the input simulating parameters, the data is generated in the form of the transmitted signal \mathbf{t} , received signal \mathbf{r} , and channel matrix \mathbf{H} along with the noise \mathbf{n} . After generating the data, it is organized to train and test the DM-Detnet model. The DM-Detnet model is integrated at the receiver end with \mathbf{r} and \mathbf{H} as input. Finally, the network model is trained and tested to obtain the output results in the form of SER. Fig. 4, presents a workflow diagram for the training and testing of the DM-Detnet scheme.

IV. SIMULATION SETUP AND RESULT DISCUSSION

In our use case scenarios, we consider ($t < r$), we assume perfect CSI, which means that we are fully aware of the channel's behavior. We employ the Rayleigh fading channel model for indoor industrial use case between t and r , where the signal goes through random fluctuations following the Rayleigh distribution. To map the information we consider the 16-QAM modulation scheme. To model the proposed DM-Detnet scheme, a MATLAB-based in-house simulator is used, that operates on an Optiplex Intel(R) processor core (TM) i9-10900K CPU @ 3.70GHz 3.70 GHz, with RAM of 128 GB (128 GB applicable), and 32 GB GPU Processor. Research simulations are conducted over a dedicated computer. A detailed summary of the simulation setup utilized for developing the DM-Detnet scheme is presented in Table 1.

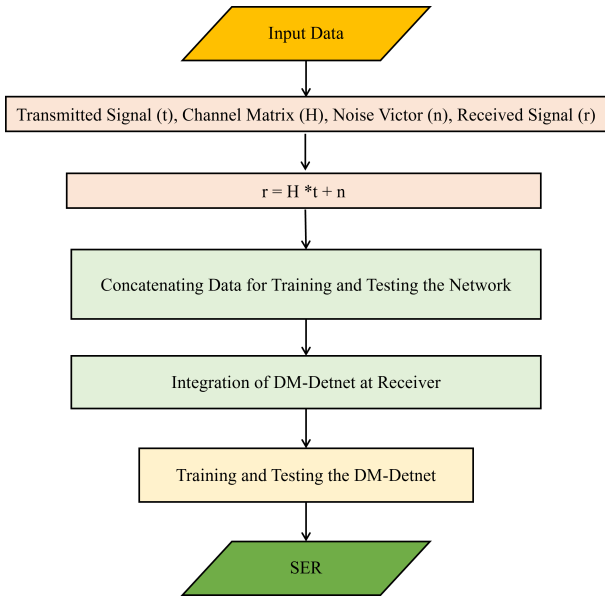


Fig. 4: Workflow chart of DM-Detnet scheme.

TABLE I: The simulation setup for the DM-Detnet scheme

| Simulation Parameters | Values |
|-----------------------------------|-----------------------------------|
| Transmitter | 2, 4, 8 and 16 |
| Receiver | 2, 4, 8 and 32 |
| Modulation Scheme | 16-QAM |
| Data Size | 1-million to 4-million |
| Training-SNR | 0:2:20-dB and 0:2:4-12:14:16-dB |
| Testing-SNR | 0:2:20-dB |
| Training Activation Function | SCG |
| Optimizers | RmsProp |
| Neurons/Layer | 64, 32, and 16 (layer 1, 2 and 3) |
| Performance Function | Cross-entropy |
| Number of Epochs | 2000 |
| Batch Size | 512 |
| Training Percentage | 70% |
| Testing and Validation Percentage | 30% |

A. Performance Evaluation of DN-Detnet against Conventional MIMO Detectors

To analyze the performance of the DM-Detnet scheme, a detailed simulation study is conducted against the conventional MIMO detectors. This simulation analysis is carried out for two different configuration MIMO scenarios across different SNR levels. Fig. 5, represents the simulation results for the DM-Detnet scheme against the conventional MIMO detectors for (a) 2x8 MIMO system and (b) 4x16 MIMO systems. Fig. 5(a) for 2x8 MIMO scenario, simulation results show that the proposed DM-Detnet scheme initially outperforms conventional MIMO detectors and shows better SER performance than the conventional MIMO detectors; however, its SER performance matches with the conventional MIMO detectors at 14dB and 16dB SNR respectively. However, beyond 14dB to 16dB SNR, the SER performance for the proposed DM-Detnet scheme saturates against conventional MIMO detectors. This saturation of the DM-Detnet scheme for 2x8 calls for additional optimization, especially training on specified SNR

values. For Fig. 5 (a) 2x8 MIMO scenario, the model was trained and tested over the complete set of SNR values ranging between 0 to 20 dB SNR. However, for the Fig. 5 (b) 2x8 MIMO scenario case, the simulation results show that optimizing the proposed DM-Detnet scheme by training over a specific SNR point (10 to 20 dB-SNR) and testing the network model over 0 to 20 dB SNR gives us optimal results by beating the benchmark conventional MIMO detectors. This is because of the large data for training over a limited number of SNR points which results in data division over specific SNR points instead of considering all SNR points which consumes less training data per point. Similarly, in Fig. 5 (c) for the 4x16 MIMO scenario, simulation results show the same trend that initially, that the DM-Detnet scheme outperforms conventional MIMO detectors and shows better SER performance than the conventional MIMO detectors, however, beyond 13 dB to 14 dB SNR, the SER performance for the proposed DM-Detnet scheme saturates against conventional MIMO detectors. Fig. 5 (d) for the 2x8 MIMO scenario, presents that the optimized DM-Detnet simulation results by increasing the data up to 4 million, training the DM-Detnet network model in two SNR divisions, specifically 0 to 4 dB-SNR and 12 to 16 dB SNR and testing the network model for 0 to 20 dB SNR. The optimized simulation results show that the proposed DM-Detnet scheme shows better results than the un-optimized DM-Detnet scheme presented in Fig. 5 (c). This study also shows that increasing the data size and considering specific low and high SNR points for DM-Detnet training results in a better performance than the un-optimized DM-Detnet model. However, it still needs more optimization and pruning of the AI model to outperform conventional MIMO detectors at high SNR points. This is currently an ongoing research study and research investigation is still in process, which is our future direction.

B. Computational Complexity

The complexity can be calculated by the total number of floating-point operations (FLOPs), which stand for computations utilizing floating-point numbers. The computational complexity of ZF and MMSE detectors mainly depends on the number of transmitted and received antennas. However, on the other hand, the complexity of MLD relies on the modulation constellation size and the number of received antennas and transmitted antennas respectively [17]. The proposed DM-Detnet scheme is based on the DNN algorithm, whose computational complexity relies on the network architecture. The computational complexity of MIMO detectors in terms of number flops is summarized in Table 2. Fig. 6, shows that the computational complexity measurements are based on the number of flops. Due to light network architecture, the DM-Detnet scheme has lower computational complexity than the benchmark conventional MIMO detectors.

V. CONCLUSION AND FUTURE RESEARCH DIRECTION

The advancement in AI has transformed MIMO technology into the most crucial technology for beyond 5G and 6G networks. However, signal detection at the receiver end becomes

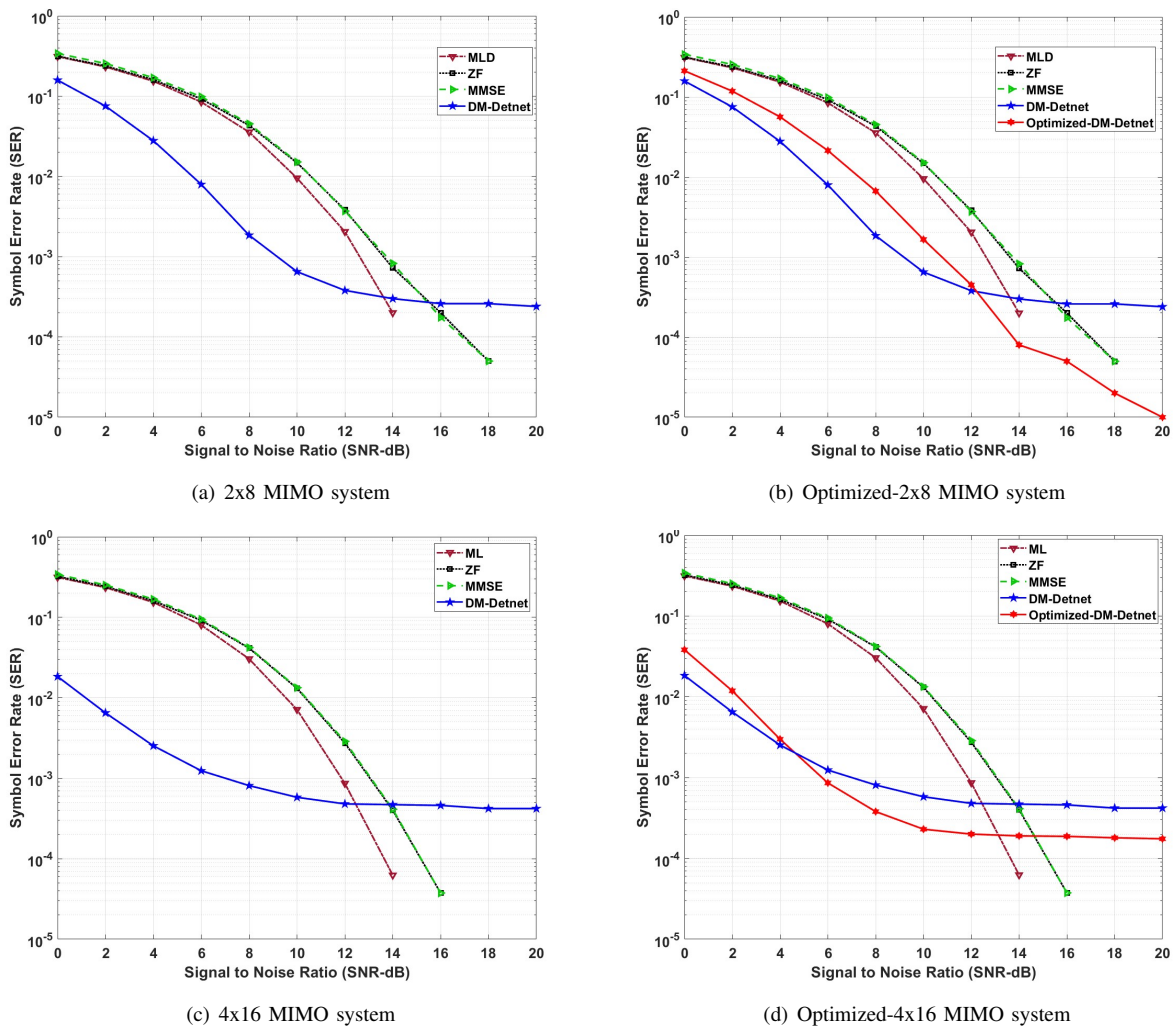


Fig. 5: Performance analysis of DM-Detnet for 2x8 and 4x16 for MIMO systems against conventional MIMO detectors.

TABLE II: Computational complexity based on No of flops

| MIMO Detector | No of Flops |
|---------------|---|
| MLD | $ m ^t * (r + 1)$ [18] |
| ZF | $14/3t^3 + 2t^2$ [19] |
| MMSE | $26/3t^3 + 4t^2$ [19] |
| DM-Detnet | $\sum_{n=3}^N N_{n-1} N_n$ (Where N_n = number of neurons in the nth layer) |

a very challenging task in MIMO technology. To address this signal detection problem, this work presents a deep MIMO detection network called DM-Detnet. The proposed DM-Detnet is an Artificial Intelligence-based MIMO detector that uses AI to detect signals for MIMO systems. In comparison to the benchmark traditional MIMO detectors, the proposed DM-Detnet is trained, tested, and optimized over specific SNR points and data size to provide optimal results for MIMO scenarios. Results show that by optimization, the proposed DM-Detnet scheme performs well against the conventional MIMO detectors and also has lower computational complexity.

However, for the 4x16 MIMO scenario at high SNR points, the DM-Detnet scheme performance saturates. This opens a new future direction for the proposed DM-Detnet scheme where future studies will concentrate on DM-Detnet network model optimization and pruning for Ma-MIMO systems.

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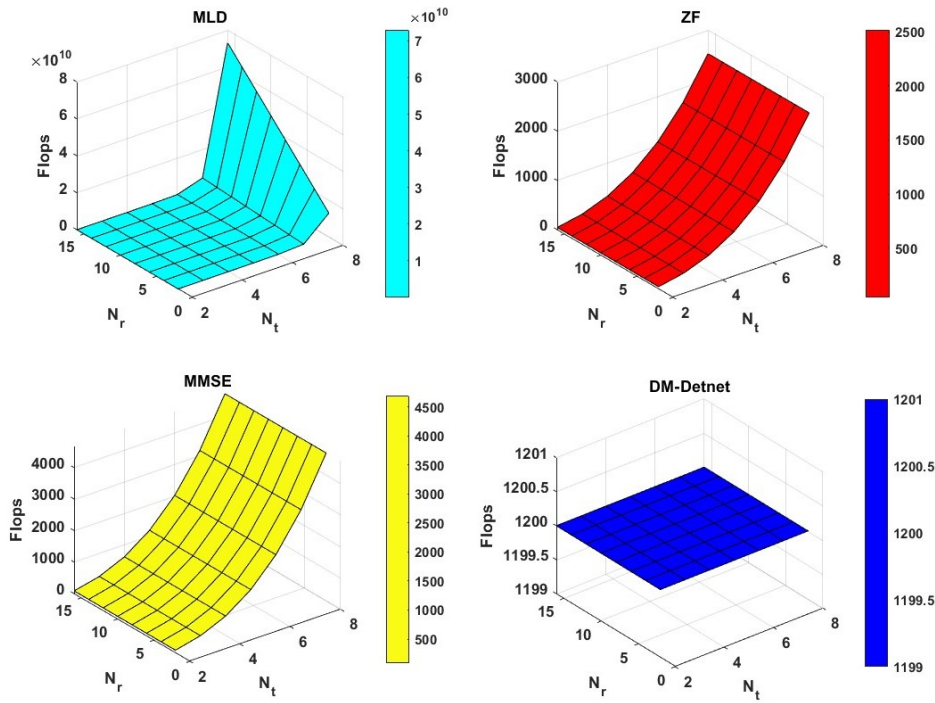


Fig. 6: Computational complexity of DM-Detnet against conventional MIMO detectors.

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