Recurrent Quantum Neural Network based EEG signal denoising for a Brain-Computer Interface

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***Abstract*—Brain-computer interface (BCI) technology is a means of communication that allows individuals with severe movement disability to communicate with external assistive devices using the electroencephalogram (EEG) or other brain signals. This paper presents an alternative neural information processing architecture using Schrödinger wave equation for enhancement of the raw EEG signal. The raw EEG signal obtained from the motor imagery (MI) of a BCI user is intrinsically embedded with non-Gaussian noise while the actual signal is still a mystery. The proposed recurrent quantum neural network (RQNN) is designed to filter such non-Gaussian noise using an unsupervised learning scheme without making any assumption about the signal type. The proposed learning architecture has been modified to do away with the Hebbian learning associated with the existing RQNN architecture [1] as this learning scheme was found to be unstable for complex signals such as EEG. Besides this the soliton behaviour of the non-linear Schrödinger wave equation was not properly preserved in the existing scheme. The unsupervised learning algorithm proposed in this paper is able to efficiently capture the statistical behaviour of the input signal while making the algorithm robust to parametric sensitivity. This denoised EEG signal is then fed as an input to the feature extractor to obtain the Hjorth features. These features are then used to train a Linear Discriminant Analysis (LDA) classifier. It is shown that the accuracy of the classifier output over the training and the evaluation datasets using the filtered EEG is much higher compared to that using the raw EEG signal. The improvement in classification accuracy computed over nine subjects is found to be statistically significant.**

I. INTRODUCTION

BRAIN-COMPUTER INTERFACE (BCI) technology is a means of communication that allows individuals with severe movement disability to communicate with external assistive devices using the electroencephalogram (EEG) or other brain signals. A typical BCI scheme is shown in Fig. 1 which consists of signal acquisition, pre-processing, feature extraction, classification and feedback as well as device commands. Using the classifier output, a control command is issued to the intended devices. Accurate signal processing (i.e. preprocessing, feature extraction and classification) is fundamental for an efficient BCI system [2] [3]. This paper focuses on the key aspect of EEG signal pre-processing for better feature extraction leading to improved classification performance.

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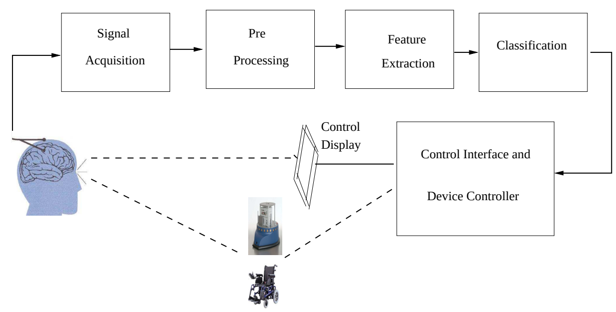

Fig. 1. Basic Design of a simple BCI system

A BCI can be based on one or more of the following cognitive events or processes: Motor Imagery (MI), P300 (event related potential) [4] [5], Visually Evoked Potential (VEP), slow cortical potentials (SCP) or activity of a single neuron (invasive). In MI based BCIs, the intentions of the subject is translated into a control signal by classifying the specific EEG pattern which is characteristic of the imagined task e.g., movement of the hand and/or foot. This paper focuses its work on the synchronous MI based BCI. Many approaches have been developed for filtering EEG. In this work our purpose is a quantum approach. Busemeyer et al.[6] analyzed the dynamics of human decision-making and showed that a better fit of the data could be achieved using quantum dynamics. Thus the Recurrent Quantum Neural Network (RQNN) model based on the novel concept that a quantum object mediates the collective response of a neural lattice [7][1] has been investigated here as a denoising mechanism in the pre-processing of the EEG signal for a synchronous MI based BCI so as to improve the feature extraction and classification processes. A similar technique was reported in [8][9] for EEG signal filtering where the error signal was used to stimulate the radial basis function (RBF) network and the weights of the network were updated using the well-known Hebbian learning rule. The new RQNN model presented here has the RBF network stimulated directly from the raw EEG signal. In addition, the learning rule for the weight updation process is different. As we will see later, this has led the model to become far more stable and become more independent of the parameters. A similar approach has been implemented successfully in many practical applications such as robot control [10], eye tracking

[1] and stock market prediction [11].

The remainder of the paper is organized into five sections. Section 2 describes the theoretical concepts of the RQNN model. Section 3 describes the methodology for processing the raw EEG signal using the RQNN modelling approach. Section 4 describes the datasets and the feature extraction methodology utilized in this work. Results are presented and discussed in section 5. Section 6 concludes the paper.

II. EARLIER RQNN MODEL REVISITED

Quantum mechanics is extremely successful in describing nature[12]. According to Bucy [13], every solution to a stochastic filtering problem involves the computation of a time varying probability density function (*pdf*) on the state space of the observed system. Dawes [14] proposed a novel model - a parametric avalanche stochastic filter using this concept. This work was improved by Behera et al.[7] by using Maximum Likelihood Estimation (MLE) instead of an inverse filter in the feedback. Further, Ivancevic [11] provided the analytical analysis of the non-linear Schrödinger equation (NLSE) and used the closed-form solution for the concerned application. Since the RQNN approach does not make any assumption about the nature and shape of the noise that is embedded in the signal to be filtered the approach is most suitable for those signals where the characteristics of the embedded noise is not known. EEG signals are one of these signals where the characteristics of the embedded noise is not known and hence this proposed work on EEG signal filtering is strongly inspired by these works.

A conceptual framework of the functioning of the RQNN model is given in Fig. 2. It is basically a one dimensional array of neurons whose receptive fields are excited by the signal input y reaching each neuron through the synaptic connections. The neural lattice responds to the stimulus by actuating a feedback signal $\hat{y}$ back to the sensory input. Intuitively, the brain can be thought of as a hierarchical system where its mental part, modeled as a quantum process, observes the unified response of a specific neural lattice and actuates a feedback signal. It has been shown that a quantum process models the average behavior (collective response) of a neural lattice [7], [15]. Thus the RQNN ignores the individual neuronal dynamics and provides a path-breaking framework to understand the unified behavior of the human brain to various types of stimuli within the scope of unity in perception defined as the binding problem in the neuroscience community [1]. The time evolution of this average behavior ψ is described by the Schrödinger wave equation (SWE) [16]:

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi(x,t) + V(x,t)\psi(x,t) \qquad (1)$$

Here, $2\pi\hbar$ is the Plank's constant, $\psi(x,t)$ is the wave function associated with the quantum object at space-time point (x,t), m is the mass of the quantum object and $V(x,t)$ is the potential field. Symbols such as i and ∇ carry their usual meaning in the context of the SWE. The $\psi(x,t)$ function represents the solution of this equation (1).

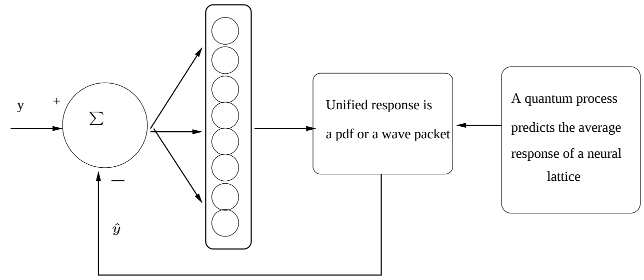

Fig. 2. Conceptual framework of RQNN

Here the neuronal lattice sets up a spatial potential field $V(x)$. A quantum process described by the quantum state ψ which mediates the collective response of a neuronal lattice evolves in this spatial potential field $V(x)$ according to (1). As $V(x)$ sets up the evolution path of the wave function, any desired response can be obtained by modulating the potential field properly.

The authors of [7] have used the RQNN for stochastic filtering. Although their filter is able to reduce noise, but its stability is highly sensitive to model parameters, owing to which, in case of imperfect tuning, the system fails to track the signal and its output saturates to absurd values. In the proposed architecture (Fig. 3), the RBF network is excited by the input signal $y(t)$. The difference between the output of the RBF network and the *pdf* feedback $|\psi(x,t)|^2$ is weighted by a weight vector $W(x)$ to get the potential field. The model can be seen as a Gaussian Mixture Model (GMM) estimator of the potential field with fixed centres and variances, and only the weights being variable. These weights can be trained using any learning rule.

The proposed filter's parameters can be tuned very easily and imperfect tuning leads only to underfiltering or overfiltering, without making the system unstable. The next section details the EEG signal filtering using the RQNN model.

III. PROPOSED RQNN MODEL FOR PRE-PROCESSING

This section describes the proposed new RQNN architecture and the complete signal processing methodology that has been implemented in the proposed BCI design. As shown in Fig. 3, the raw EEG signal is fed to the RQNN and a denoised signal is obtained. This raw EEG signal is first normalized in the range 0-2 before it is fed to the RQNN filter.

During the offline training process, the complete set of the normalized EEG data (here signals from channels C3 and C4 discussed in the next section) is fed through the RQNN to obtain the filtered EEG as shown in Fig. 4. Here the EEG data from the C3 and the C4 channel is fed to the two RQNNs respectively and a filtered estimate of the signal is obtained for the samples from both these channels. The next task is to obtain the Hjorth features [17] from this RQNN filtered EEG signal. These Hjorth features are then fed as an input to train the offline classifier which in this case is linear discriminant analysis (LDA). Once the offline analysis is complete and the LDA classifier is trained, the parameters/weight vector

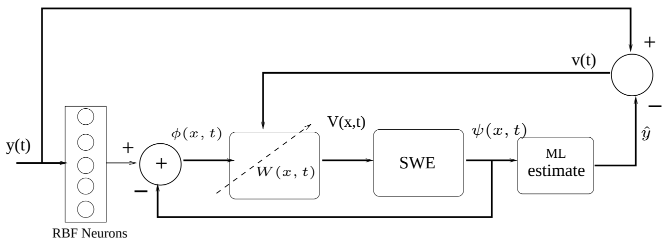

Fig. 3. EEG signal filtering using RQNN model

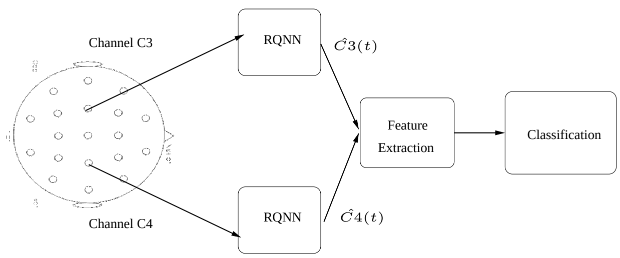

Fig. 4. Framework of the EEG signal filtering using RQNN model

is stored for use with the classifier to identify the unlabeled EEG data during the online analysis. Here it needs to be clarified that in order to capture the dynamic property of the continuous EEG signal, the weight updation process of the RQNN filter is continuous (to de-noise the EEG signal) during both the offline and the online process while the classifier parameters of the LDA are tuned offline and then kept fixed during the online classification process.

In the RQNN, we make the hypothesis that the average behaviour of a neural lattice that filters the EEG signal is a time varying *pdf* which is mediated by a quantum object placed in the potential field and modulated by the input EEG signal so as to transfer the information about the *pdf*. We use the SWE to track this *pdf* since it is a well-known fact that the square of the modulus of the ψ function, the solution of this wave equation, is also a *pdf* (denoted hereby as $\rho(.)$).

The potential field is calculated as below:

$$V(x,t) = \zeta W(x,t)\phi(x,t) \tag{2}$$

where,

$$\phi(x,t) = exp(\frac{-(x-y(t))^2}{2\sigma^2}) - |\psi(x,t)|^2 \tag{3}$$

where $y(t)$ is the input signal and the synapses are represented by time varying synaptic weights $W(x,t)$.

This potential field modulates the non-linear SWE described by the equation (1).

The filtered estimate is calculated as:

$$\hat{y}(t) = \int x\rho(x,t)dx \tag{4}$$

Based on this estimate, the weights are updated and thus establishing a new potential field for the next time evolution.

When the estimate $\hat{y}(t)$ is the actual signal, then the signal that generates the potential field for the SWE, $\hat{\nu}(t)$, is simply the noise that is embedded in the signal. If the statistical mean of the noise is zero, then this error correcting signal $\hat{\nu}(t)$ has little effect on the movement of the wave packet. Precisely, it is the actual signal content in the input $y(t)$ that moves the wave packet along the desired direction which in effect achieves the goal of the EEG signal filtering. It is expected that the synaptic weights evolve in such a manner so as to drive the ψ function to carry the exact information of the *pdf* of the filtered EEG signal $\hat{y}(t)$. To achieve this goal the weights are updated using the following learning rule.

$$\frac{\partial W(x,t)}{\partial t} = -\beta_d W(x,t) + \beta\phi(x,t)(1+\nu(t)^2) \tag{5}$$

where β is the learning rate and β_d is the de-learning rate. De-learning is used to forget the previous information, as the input signal is not stationary, rather quasi-stationary in nature.

A. Numerical Implementation

The space variable x is defined uniformly spaced as $x_n = n\delta x, n = 1,...,N$ and the time is spaced as $t_k = k\delta t, k = 1,...,T$. The potential function is approximated as $V(x_n,t_k) = V_n^k$. This potential function excites the NLSE to obtain the quantum wave function ψ_n^k. Various methods, both explicit as well as implicit, have been developed for solving the NLSE numerically, on a finite dimensional subspace [18]. We implemented two schemes. Firstly, we used Crank Nicholson implicit scheme for solving the NLSE which requires a quasi tridiagonal system of equations to be solved at each step. This scheme, although accurate, requires to solve for the inverse of a huge $N \times N$ matrix, which is time consuming. Hence we implemented the same using the explicit scheme.

$$i\frac{\psi_n^{k+1} - \psi_n^k}{\delta t} = -\frac{\psi_{n+1}^k - 2\psi_n^k + \psi_{n-1}^k}{2m\delta x^2} + V_n^k\psi_n^k \tag{6}$$

This method is linearly stable for $\delta t/(\delta x)^2 \leq 1/4$, with a truncation error of the order of $(O(\delta t^2) + O(\delta x^2))$ [18].

Another point to note is that we need to maintain the normalized character of the pdf envelope, $|\psi|^2$, by normalizing at every step. $\sum_{n=1}^{N} |\psi_n^k|^2 \delta x = 1$ for all k.

B. Selection of Parameters

The parameter $\hbar$ has been set as unity (The Planck constant is an atomic-scale constant. The atomic units are a scale of measurement in which the units of energy and time are defined so that the value of the reduced Planck constant is exactly one) and the other four parameters used to obtain the filtered EEG signal have been set heuristically as β = 2.7, β_d = 1, m = 0.25 and ζ = 15 for the RQNN with non linear modulation of the potential field. β is necessary to update the synaptic weight vector $W(x,t)$. ζ is the scaling factor to actuate the spatial potential field $V(x,t)$. The number of neurons along the x-axis is fixed at N = 400. In addition, the number of iterative steps that are required for the response of the wave equation to reach a steady state to any particular computational sampling instant of the EEG has been kept at 20. Thus a particular sample is iterated 20 times before the next sample is presented. The value of the weight and the potential function evolves in this loop. All the above parameters have been obtained heuristically and kept same for all the subjects.

IV. Datasets and Features

The EEG data used in this analysis is dataset 2b provided in the BCI competition IV [19] (with each subject contributing a single session referred as *03T for the training phase and two sessions referred as *04E, *05E for the evaluation phase). The 3T, 4E and 5E series datasets belong to the 3rd, 4th and 5th session trials respectively. It is assumed that the user will have gained a reasonably good control over his/her motor imagery skills after performing one feedback and one non feedback session previous to this session. The dataset is obtained using a cue-based paradigm which consists of two classes, namely the MI of left hand (class 1) and right hand (class 2). Three EEG channels (C3, Cz, and C4) were recorded in bipolar mode with a sampling frequency of 250 Hz and were bandpass-filtered between 0.5 Hz and 100 Hz, and a notch filter at 50 Hz was enabled. As shown in Fig.

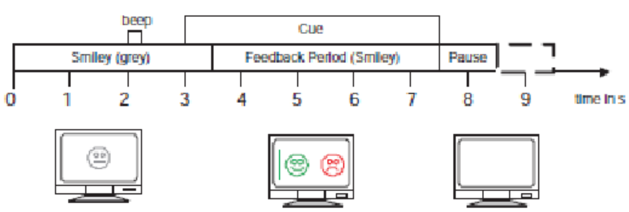

Fig. 5. Timing scheme of the paradigm with Smiley feedback [19]

5, the trial paradigm started at second 0 with a gray smiley centered on the screen. At second 2, a short warning beep (1 kHz, 70 ms) was given. The cue was presented from second 3 to 7.5. Depending on the cue, the subjects were required to move the smiley towards the left or right side by imagining left or right hand movements respectively. At second 7.5 the screen went blank and a random interval between 1.0 and 2.0 seconds was added to the trial so as to avoid user adaptation. A more detailed explanation of the EEG signal recording methodology for the training and the testing datasets is available in [19].

Various approaches have been investigated to produce a good practical BCI system. Features such as the amplitude values of the RQNN generated wavepacket, bandpower, Hjorth, power spectral density (PSD), auto regressive (AR) model, time frequency (t-f) features have been utilized by various research groups [9], [8], [20], [21], [22], [23].

Most of the BCI research in signal processing is focused in the frequency domain and the RQNN based EEG filtering is in the time domain. It is possible to convey relevant information about the EEG epochs with the trio of combinations of conventional time domain based descriptive statistics Hjorth parameters namely; activity, mobility and complexity [24]. The computational cost in the calculation of the Hjorth parameters is considered low as this approach is simple and based on the calculation of variance [25]. In addition, the Hjorth parameters, especially complexity, are sensitive to noise because their computation is based on the numerical differences and their variances [26]. This prompted the authors to evaluate the RQNN pre-processing technique by utilizing the Hjorth parameter representation for the discrimination of mental tasks as a part of this investigation.

The 1st Hjorth parameter is a measure of mean power of the signal characteristics in terms of activity (variance of the signal) and is mathematically defined as

$$Activity(y) = \sum_{i=1}^{N_S} \frac{(y(i) - \mu)^2}{N_S} \tag{7}$$

The 2nd Hjorth parameter called mobility is an estimate of the mean frequency and is defined as

$$Mobility = \sqrt{\frac{var(y')}{var(y)}} \tag{8}$$

The 3rd Hjorth parameter called complexity is an estimate of the bandwidth of the signal and is defined as

$$Complexity = \frac{Mobility(y')}{Mobility(y)} \tag{9}$$

For (7) to (9) above, y is the signal, y' is the first derivative of the signal, N_s is the number of samples in the window and μ is the mean of the signal in the window.

The Hjorth features have been used as discriminative features on a 1 s time window from the two channels (C3 and C4) so as to distinguish the output classes. These features are fed to train the LDA classifier in the offline process.

V. Results and Discussion

We present the results of the proposed approach in this section. The results show that without prior knowledge of the type of the noise characteristics present in the EEG the RQNN can be utilized to enhance the information contained in the raw EEG signals (which consists of noise due to numerous factors such as artefacts, amplifiers, electrode placements) and that the quantum approach based filtering method can be used as a signal pre-processing method for a more efficient MI based EEG signal classification. A clear improvement is observed in terms of the classification accuracy (CA) and the confidence or strength of this accuracy. This improvement is with reference to a non filtered or raw EEG signal while the same Hjorth features and the LDA classifier is used i.e., the only change is the presence or absence of the RQNN filter.

Fig. 6 displays the representative plot of the raw EEG signal and the RQNN filtered EEG signal for the subject 0403T for a time interval between 5 to 6 sec. Corresponding to this filtered EEG Fig. 7 displays the tracking of this signal in the form of snapshots of the wave packets. Here, the wave packet slides along the X axis with time at t = 5.2s, t = 5.6s and t = 6.0s.

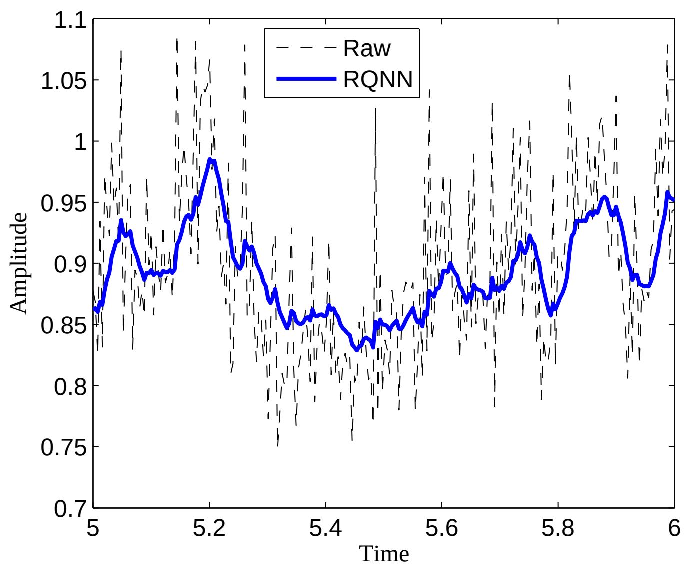

Fig. 6. Representative plot of the Raw EEG and the RQNN filtered signal

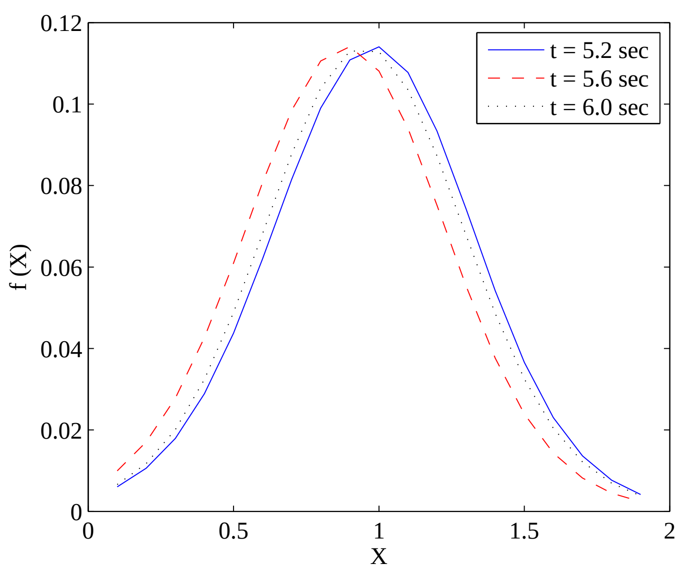

Fig. 7. Snapshots of the wavepackets corresponding to the representative plot of the RQNN filtered EEG signal shown in Fig. 6

Fig. 8 and Fig. 9 display the CA plot using the LDA classifier with the Hjorth features generated using the raw EEG signal and the RQNN filtered EEG signal for the training dataset of the subject 0403T and 0803T.

A very important method to indicate the result and the certainty of classification is that of time-varying signed distance

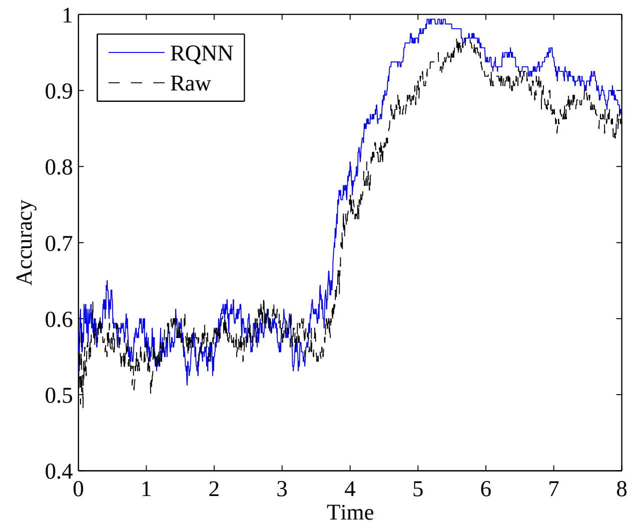

Fig. 8. CA plot for the subject 0403T on the raw EEG and the RQNN filtered EEG using the LDA classifier

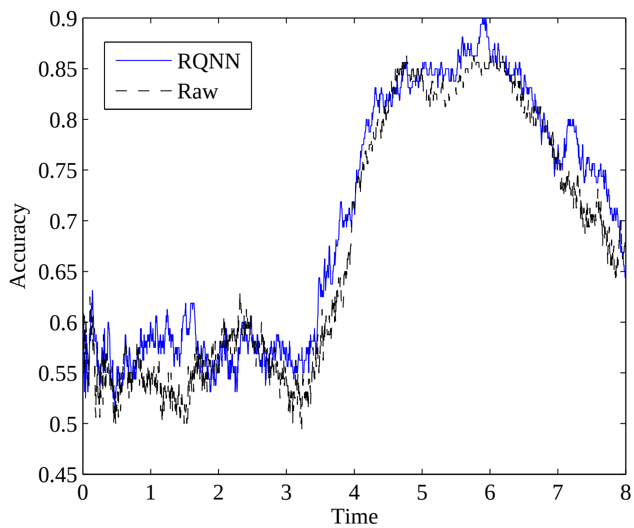

Fig. 9. CA plot for the subject 0803T on the raw EEG and the RQNN filtered EEG using the LDA classifier

(TSD) obtained as the output of the LDA classifier. The sign of classification indicates the result i.e., the class to which a sample belongs and the magnitude of the classification indicates the confidence in the classification. Fig. 10 displays the mean TSD for the subject 0403T during the training phase. It is very clear from this figure that in addition to the CA, the strength or the confidence of the classification also shows improvement with the RQNN filtered signal compared to that obtained using the raw EEG signal.

Once the classifier is trained, the parameters obtained are stored for use in the on-line classification with the test data. Appropriate bias adjustment has been made for both

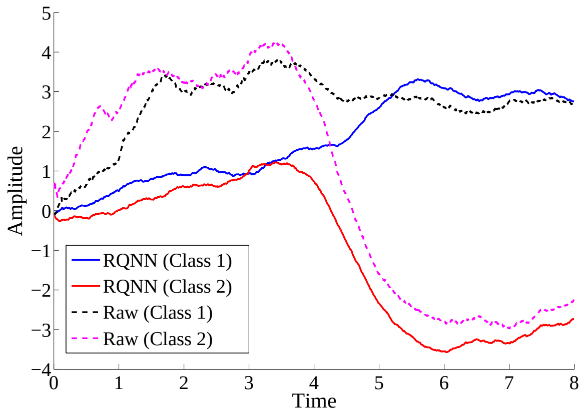

Fig. 10. TSD plot using the LDA classifier on the training dataset using the Raw EEG and the RQNN filtered EEG for the subject 0403T

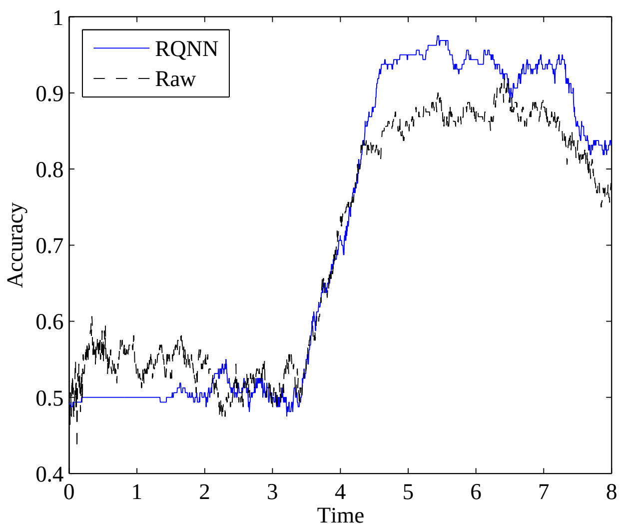

Fig. 11. CA plot for the subject 0404E on the raw EEG and the RQNN filtered EEG using the LDA classifier

the RQNN filtered data and the raw data. Fig. 11, 12, 13 and 14 display the CA plot with the evaluation/unlabelled EEG dataset using the LDA classifier. Thus, as can also be inferenced from the figures, the performance in terms of CA shows substantial improvement with the RQNN filtering approach on the evaluation dataset as well.

The RQNN filtering approach discussed above has been investigated for all the nine subjects. The value of maximum of CA and Kappa are given in the Table I for the training and evaluation dataset with and without the use of the RQNN filter.

It can be seen from the results displayed in the Table I that the RQNN improves the CA for eight out of nine subjects during both the training and the evaluation stages. In addition, the maximum of the kappa value also shows a significant improvement with the use of the RQNN filtering

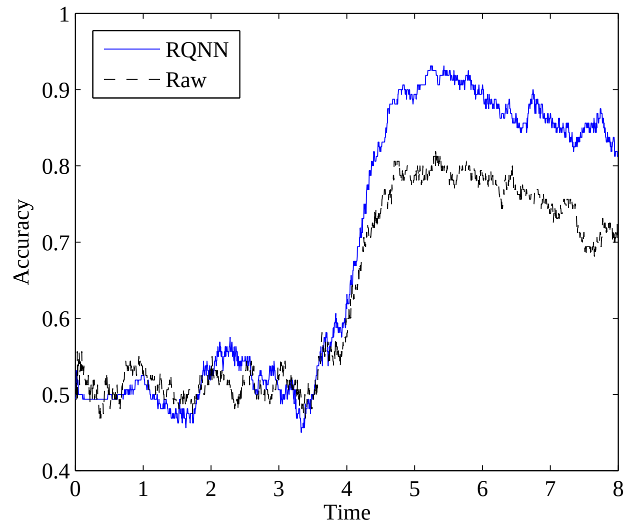

Fig. 12. CA plot for the subject 0405E on the raw EEG and the RQNN filtered EEG using the LDA classifier

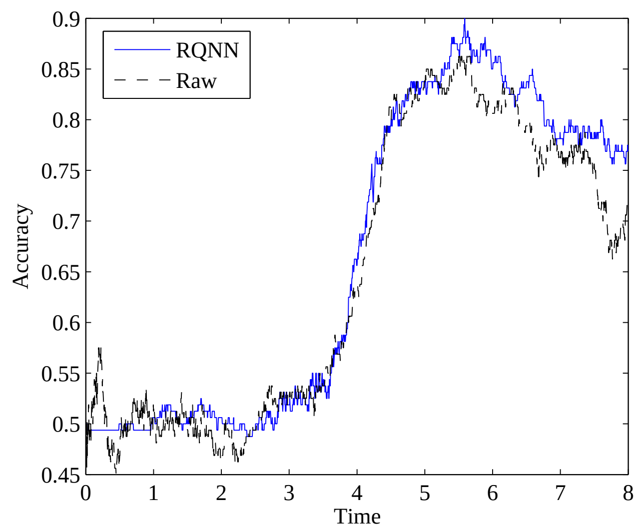

Fig. 13. CA plot for the subject 0804E on the raw EEG and the RQNN filtered EEG using the LDA classifier

technique. The mean CA over all the nine subjects indicates that the RQNN improves the CA by a margin of about 5% for both the training and the evaluation stages. Similarly, mean of the maximum of the kappa value for all the nine subjects also shows an overall improvement of 0.1 for both the training and evaluation stages. The maximum of the kappa value from both the evaluation stages shows an improvement from the value of 0.54 to a value of 0.63, which is higher than the value of 0.6 (obtained by the winner of the BCI competition). To determine if these improvements are statistically significant, two-way Analysis of Variance (ANOVA2) test has been performed with the results of the

TABLE I
COMPARISON IN TERMS OF CA USING THE LDA CLASSIFIER WITH AND WITHOUT THE RQNN FILTER

Subj.	Training (03T)				Evaluation (04E)				Evaluation (05E)				Evaluation Stages	
	RQNN		Raw		RQNN		Raw		RQNN		Raw		RQNN	Raw
	Max Acc.	Max Kappa	Max Acc.	Max Kappa	Max Acc.	Max Kappa	Max Acc.	Max Kappa	Max Acc.	Max Kappa	Max Acc.	Max Kappa	Max Kappa	Max Kappa
B01	81.87	0.64	81.87	0.64	77.50	0.55	71.25	0.42	65.62	0.31	63.75	0.26	0.55	0.42
B02	79.37	0.59	66.87	0.34	66.66	0.33	63.12	0.27	61.25	0.22	58.12	0.16	0.33	0.27
B03	75.00	0.65	69.37	0.39	77.50	0.55	63.75	0.28	80.62	0.61	60.00	0.20	0.61	0.28
B04	99.38	0.99	96.87	0.94	97.50	0.95	91.87	0.84	93.13	0.86	81.87	0.64	0.95	0.84
B05	73.12	0.46	78.12	0.56	80.62	0.61	89.37	0.79	66.87	0.34	85.00	0.70	0.61	0.79
B06	75.62	0.51	73.75	0.47	78.13	0.56	70.00	0.40	80.62	0.61	71.25	0.42	0.61	0.42
B07	90.63	0.81	70.00	0.40	68.12	0.36	63.12	0.26	70.00	0.40	64.38	0.29	0.40	0.29
B08	90.00	0.80	86.25	0.72	90.00	0.80	86.25	0.72	96.87	0.94	90.62	0.81	0.94	0.81
B09	88.75	0.77	87.50	0.75	86.25	0.72	86.88	0.74	83.12	0.66	83.12	0.66	0.72	0.74
Mean	83.74	0.69	78.95	0.57	80.25	0.60	76.17	0.52	77.56	0.55	73.12	0.46	0.63	0.54

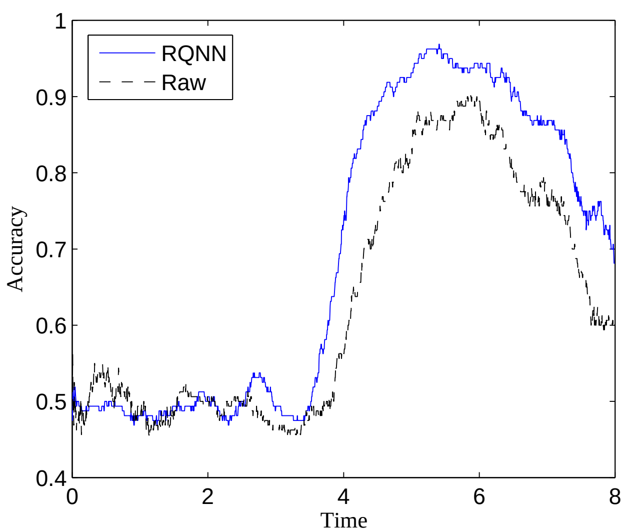

Fig. 14. CA plot for the subject 0805E on the raw EEG and the RQNN filtered EEG using the LDA classifier

training and the evaluation stages for the RQNN filtered and the raw EEG approach. The p-values of 0.0074 and 0.0057 were obtained for the accuracy and the kappa respectively. As these p-values are much less than 0.05, we can reject the null hypothesis and accept the alternative hypothesis that the results from the RQNN filtered EEG are likely to be different from the result of the raw EEG i.e., the improvements due to the RQNN filter based technique are statistically significant.

It needs to be emphasized here that the parameters for the RQNN have not been tuned but have been heuristically found and the values are kept the same for all the nine subjects. It is well known that the parameters chosen for any network on an EEG of one subject will be different from the parameters for the same network on another subject. This makes the above results even more encouraging since there is no need to tune the filter parameters for every subject.

Let us now discuss the subject 0503T which had not shown improvement with the heuristically obtained parameters. We have obtained parameters for this subject using the Particle Swarm Optimization (PSO) technique [27]. The Fig. 15 displays the CA plot for this subject with the raw EEG, the heuristically obtained parameters and with the PSO tuned parameters. As can be seen from this figure, the peak CA with the PSO tuned parameters is 91.87%. The maximum of the kappa value is found to be 0.84. The results for this subject show a very good improvement by utilizing the optimization technique. Hence future work will involve evaluating the results for this novel approach after different subject dependent parameters have been obtained for the RQNN using appropriate optimization techniques.

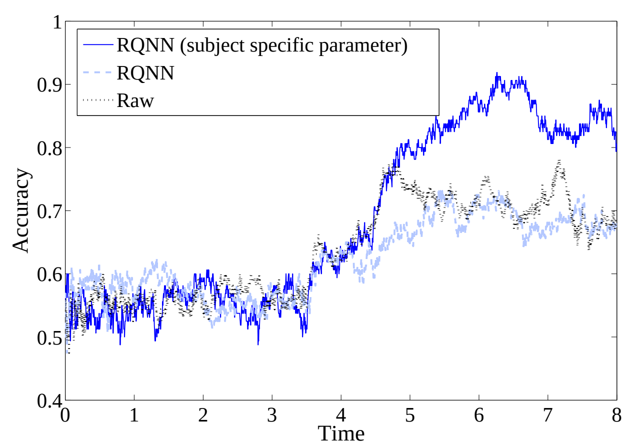

Fig. 15. CA plot for the subject 0503T on the raw EEG and the RQNN filtered EEG (on subject independent and PSO tuned subject specific parameter) using the LDA classifier

VI. CONCLUSION

In this paper, the raw EEG data has been pre-processed using the recurrent quantum neural network. The learning architecture and the associated unsupervised learning algorithm have been modified to take into account of the complexity of the EEG signal. The basic approach is to ensure that the statistical behavior of the input signal is properly transferred to the wave packet associated with the response of the quantum dynamics of the network. At every computational sampling instant, the EEG signal is encoded as a wave packet which can be interpreted as the *pdf* of the signal at that instant. Features are then extracted form the enhanced EEG data using the well-known Hjorth approach. The CA based on these features on the EEG signal shows an overall improvement of approximately 5% during both the training and the evaluation phases. This is quite a significant result. At this stage it is very important to note that these results are obtained without any parameter tuning. It is quite well-known that there are always subject dependent parameters [28], [29] and hence it seems as of now that the results can be greatly improved by selecting more suitable parameters. This is the future scope of this work. Simultaneously it should also be noted that these parameters associated with the nonlinear SWE have been determined in this work in a heuristic way. These parameters can be properly optimized to provide better accuracy (as is also verified from the case of the subject 0503T shown in Fig. 15). In fact we believe that each subject can be associated with an optimized parameter set consisting of m and ζ. Hence we plan to use PSO or genetic algorithm (GA) [30] to select near-optimal parameters for each subject. Towards this end, the investigation is currently underway.

The remarkable feature of the proposed scheme is that without introducing complexity in the classification layer, the classification accuracy can be improved simply by enhancing the EEG signal.

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