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OCTONIONS, THE THREE FLAVOURS OF MATTER AND A NEW KIND OF SUPER-SYMMETRY

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ABSTRACT

It has been long theorized that there is some relationship between Octonions and the Standard Model. This paper reveals that there is just such a relationship and that it can solve many of the questions surrounding the Standard Model. These include: the seeming disorder of the particles and their masses; why there are three flavors of matter; and why some bosons are massless and some aren't. The solution comes from the precise ordering of the elementary particles in the Pentonions, before expansion via triangular numbers into the Octonions, leading to a potentially new kind of Super-Symmetry.

Keywords: Quaternions, octonions, sedenions, pentonions, trionions, standard model, elementary particles, super-symmetry.

INTRODUCTION

The Standard Model was introduced in the 1970s, as a means of classifying and categorizing the increasing number of particles that were being detected via experiments. Particles like the 'Lambda', 'Sigma', and XI particles (Verwiebe *et al*., 1962). There were so many in fact that physicists were running out of names for the particles. It was deemed necessary to put some order on what was being dubbed 'a particle zoo'. Physicists and theorists were able to achieve this by grouping particles into categories like leptons, quarks, and gauge bosons, and further classifying them by their charges, spins, and masses. This led to the current standard model with its roughly 32 particles that we know today.

However, even with this, many physicists are not satisfied. It is true that the particle zoo has been put in order. However, the order itself is not entirely regular and leaves us with many questions. Why do the particles have those particular masses? Why are there exactly three generations of quark? Why do some gauge particles have mass and others none? Why are 2nd and 3rd generation quarks unstable, while the corresponding leptons aren't? Why don't leptons and quarks interact directly? Why is there more matter in the Universe than anti-matter? And so on.

Many attempts have been made to formulate answers to these questions. One of the most prevalent is the Representation Theory. This theory was first proposed by Wigner (1959), who has noted an important relationship between the Group Theory and the particles in the Standard Model.

However, there is another less prevalent solution, which is given by the Octonions. To date, there have been many attempts to fit the particles of the Standard Model – along with their various spins and charges – into the division algebras that describe the octonions. There are numerous researchers, both amateur and professional investigating this possibility. One of the most high-profile examples is the work by Furey (2018) that relies on complicated mappings between the Octonions and Clifford Algebras.

There is something admittedly satisfying about attempting to create a table that brings order to the disparate elementary particles in the Standard Model. The author has spent long hours attempting just this – making use of both octonions and sedenions– without much success.

That was until, the Δ and !Δ multiplication of the quaternions were applied and things appeared to pop into place (O'Neill, 2021). It is now possible to describe nearly all of the particles in the Standard Model, with one simple table.

The method came to the author, while pondering the submatrices of the 5-dimensional Higgs Boson. There are two 'submatrices'; the quaternions and the trionions. The trionions and pentonions are themselves submatrices of the octonions. Therefore, the 3 matrices – using programming language – can be written as follows:

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Pentonions = [1:5, 1:5]
Quaternions = [:4, :4]
Trionions = [1:4, 1:4]

There is clear and expected overlap here. The first attempt at structuring the particles into their respective submatrices can be seen in Figure 1. This, as we can see,

g	d	c	t	H^1
u	y	ve	μ	H^2
s	e	y	$v\tau$	H^3
b	$v\mu$	τ	y	H^4
H^1	H^2	H^3	H^4	G

Fig. 1. The Pentonionmatrix 1.

contains all of the particles of the Standard Model, excepting the *W* and *Z* bosons. We also see that there appear to be 8 Higgs Bosons. This is misleading however, as we shall see.

If we flip this table around by its anti-diagonal axis – in other words, we make the blue column the axis of multiplication – we see that wherever an 'H' and an 'H' multiply along the diagonal it forms a massless boson. In all other instances they make a massive particle. This is very similar to what we see in the Kronecker Delta, where a similar value by a similar value equals to '1' and otherwise equals to '0'. In this case, it is the other way around. This is a traceless or massless trace matrix. This not only makes it very similar to the traceless Hermitian matrices used in the Gauge Theory. It is also suggestive of a Real numbered matrix, which is very promising.

The only value that is speculative is the Graviton, as we do not have experimental evidence for its existence. However, the Graviton is theorized to have zero mass, so it definitely fits the pattern. It is interesting that it should be the Higgs that determines the massless bosons. However, when we consider that Figure 1 is the embedding of an Octonion matrix, then it is clear that the massless bosons are the result of like imaginary or real terms multiplied together, just as it is in the DGO Standard Model.

There are many pluses to this table shown in Figure 1. For instance, we see that all of the gluons and quarks are arranged in the quaternion group. The leptons, as we would expect, are arranged in the Trionion group, and both the Higgs and Graviton lie outside in the Pentonions.

The Three Flavours of Matter

During the Quaternionic construction of the quarks and gluons and the Trionionic construction of the *W*/*Z* bosons and leptons, it was clear that there was no upper limit on the creation of these new particles. All we had to do was sum another particle with its companion and *hey presto!* we had another generation.

Now, we see that there is a limitation, by way of the shape of the dimensions of the matrix alone. While this explanation might certainly appear simple – and it is – it was a long road getting to this point. The reason why the explanation is so simple, has to do with the simplicity of the question involved. There is however possibly one other factor to consider.

Notice that there are three photons in this model, just as we noted in the Dionionic model. Earlier, we said that these three photons might represent different energy levels of light needed to generate the various flavors of matter. This suggests that once elementary particles reach a lower limit on their energy levels, they stop being able to form new flavors of particles, as one might expect and has been observed by experiment.

By extension, this implies that the gluon is somehow a photon that is even more energetic than gamma rays. Although this is far from certain.

Second Attempt

There are some clear problems with the table, however. For instance, where are the *W* and *Z* bosons? And worse, why aren't the pentonions arranged in the normal pentonion matrix? They appear to be superimposed on the quaternions, which is not how we originally formulated them (O'Neill, 2021), where they were the [:6, :6] submatrix of the Octonions.

This was a clear mistake. There are two ways to find a solution to this mistake. The first way, is to recalculate the Higgs particles based on the Pentonions shown in Figures 2 and 4 to see if they still give us workable results. The other way is to rejig the table itself to include the correct version of the Pentonions.

Taking the second route first, we arrive at an entirely new table (Fig. 2). One of the upshots of this model is that it allows us to add the *W* and *Z* bosons into their correct positions. This appears to work. However, we are left with what to do with the blue-cells and the Higgs bosons. The solution is the one shown in Figure 2. This creates a kind of chequerboard pattern featuring our 4 Higgs bosons and 8 Gravitons. Now, we really do have an ordered table of all the most important particles in the

Standard Model that perfectly reflects the symmetries of the DGO model.

However, there are several differences and corrections that we are forced to make. To begin with, as anyone will tell the reader, this is no longer a 5-dimensional matrix. It is clearly a 6-dimensional one. This implies that the Pentonions were actually Sextenions all this time. But then that would imply that the Trionions are really quaternions, which is true in some sense, making the quaternions what exactly? It is clear how the confusion arose.

The Trionions are made up of imaginary numbers, while the quaternions include a real number. The extension of the quaternions shown in Figure 2 are just the octonions; e_4 and e_5, which means that the rest of them are just the pentonions, as we defined them (O'Neill, 2021). So, technically, they are 5-dimensional but taken altogether the whole system is 6-dimensional – much like parts of the String Theory.

g	d	$1W1Z$	c	WZ	t
u	y	ve	μ	H^1	G
$1W1Z$	e	y	$v\tau$	G	H^2
s	$v\mu$	τ	y	H^3	G
WZ	H^1	G	H^3	G	H^4
b	G	H^2	G	H^4	G

Fig. 2. The Pentonion matrix 2.

Luckily, this shift in dimension hasn't affected several of our key assumptions or inferences. For instance, the triune nature of our principal quark and lepton flavors still holds. It is also clear that a new set of W and Z bosons (an even lighter set) can exist where the gluon is. If so, this can be indicative of the union of the Strong and Weak forces.

If anything, it looks like the Graviton is a less energetic form of the photon. But that can't be right, as the gravitons are now 5-dimensional and higher dimensions always have more energy. So, the strong-electroweak unification must wrap around to the Graviton to form the unification of the electroweak-strong force and gravity. If so, then it only relies on two of the Gravitons, i.e. the two gravitons on the trace.

Which leads us to the next set of questions. Why are there 8 Gravitons? Moreover, if only the trace of the matrix is massless, and the Graviton itself is known to be massless, then why do six of them lie off the trace?

The 8 Gravitons

If we substitute the eight Gravitons for the eight color charges of the gluons, we begin to see the pattern. This shows that there is a clear relationship between the Graviton and gluon color charge. This relationship we might well expect, as the Graviton would need access to all four fundamental forces in order to interact with the various forms of matter.

g	d	$1W1Z$	c	WZ	t
u	y	ve	μ	H^1	g_1
$1W1Z$	e	y	$v\tau$	g_4	H^2
s	$v\mu$	τ	y	H^3	g_5
WZ	H^1	g_6	H^3	g_3	H^4
b	g_2	H^2	g_7	H^4	g_8

Fig. 3. The gluon-graviton relationship.

g	d	$1W1Z$	c	WZ	t
u	y	ve	μ	H^1	G_1
$1W1Z$	e	y	$v\tau$	G_4	H^2
s	$v\mu$	τ	y	H^3	G_5
WZ	H^1	G_6	H^3	G_3	H^4
b	G_2	H^2	G_7	H^4	G_8

Fig. 4. The complete Pentonion matrix 2.

The two linearly dependent gluons g_3 and g_8 shown in Figure 3 or their graviton corollaries G_3 and G_8 shown in Figure 4 lie on the trace but none of the others do, which either suggests that those Gravitons do possess some mass or that they inherit their massless property from the trace.

for comparison with the results shown in Figure 4. While, Figure 5 shows the corresponding section of the Octonions.

This also tells us that the G_3 and G_8 may be crucial to the unification process. Perhaps, they are the linear summation of the traces of the electromagnetic U(1) and weak force SU(2) and SU(3) color charges in some way? Whatever the truth, the relationship between the Graviton and the eight flavors of gluon tells us something important about the structure of the 5D graviton itself.

Recall that all 8 of the gluon color charges are part of a single 4-dimensional rhombic-dodecahedron (or hypercube). This suggests that all eight gravitons are similarly just different aspects of a single 5-dimensional polyhedron. From this perspective, there is likely only one generation of Graviton and by extension one flavor of the Higgs. As we know, or suspect, the gluon (g_3 and g_8 included) is both massless and its own anti-particle. Therefore, all of the Gravitons inherit their 'lack of mass' from the massless trace Gravitons, which makes sense, if they are all one massless object.

1	*i*	*j*	*k*	*E*	*I*
i	–1	*k*	–*j*	*I*	–*E*
j	–*k*	–1	*i*	*j*	*K*
k	*j*	–*i*	–1	*K*	–*j*
E	–*I*	–*j*	–*K*	–1	*i*
I	*E*	–*K*	*j*	–*i*	–1

Fig. 5. The corresponding section of the Octonions, for comparison with the results shown in Figure 4.

However, the four Higgs particles (H^1, H^2, H^3, H^4) can refer to positive and negative charges of two flavored particles, in which case there are two generations of Higgs and Graviton particles. A quick comparison between our DGO Standard Model and the corresponding section of the Octonions reveals that there are between 6 or even 8 Higgs variations. Correspondences like these are fun to play around with and can in future lead to instructive relationships between the pentonions and the particles, as well as the relationships between the elementary particles and one another. There may even be a potential for a relationship between octonion multiplication and particle interactions but this remains to be seen.

Nevertheless, we can draw the following relationships:

I and $-I = H_1$, t, and b

j and $-j = \mu$, $\nu\mu$, G_4, G_5, G_6, G_7, One W and One Z bosons

This suggests that there is a relationship between the Higgs and the top and bottom quarks. This relationship has been noted elsewhere (Heppenheimer, 1994), particularly as it pertains to the top quark and the Higgs. As for the second group of particles, it looks like μ and $\nu\mu$ may transform into the G_4 to G_7 antiparticles via the One W and One Z bosons, given enough energy.

Charges, Spins and Masses

So far, we have been able to fit the particles of the Standard Model into the (6×6) Pentonions submatrix of the Octonions. This gives us a total of 34 particles (48 if to include all of the other Gravitons and Higgs particles as particles in and of themselves) and includes the antiparticles. The reason for the increase is due to the inclusion of One W and One Z bosons (Barger *et al.*, 1980).

1	1/2	1	1/2	1	1/2
1/2	1	1/2	1/2	0	2
1	1/2	1	1/2	2	0
1/2	1/2	1/2	1	0	2
1	0	2	0	2	0
1/2	2	0	2	0	2

Fig. 6. The spin matrix.

0-1	1/3	0-1	2/3	0-1	2/3
2/3	0	0	1	0	0
0-1	1	0	0	0	0
1/3	0	1	0	0	0
0-1	0	0	0	0	0
1/3	0	0	0	0	0

(a)

0.0	0.0047	50.0	1.28	91.19	173.1
0.0022	0.0	2.2e-08	0.10566	124.97	0.0
50.0	0.000511	0.0	0.0182	0.0	124.97
0.96	0.00017	1.7768	0.0	124.97	0.0
91.19	124.97	0.0	124.97	0.0	124.97
4.18	0.0	124.97	0.0	124.97	0.0

(b)

Fig. 7. (a) The charge matrix with linear regression line showing how the data splits; (b) The mass matrix.

We can take the analogies between the Pentonions and the particles further, by examining their relationship to the particle's various spins, charges, and masses. Plotting the particle spins does nothing, except confirms the same structure we had before, which is indicative of being on the right track (Fig. 6). Figure 6 is obviously based on the assumption that the Graviton has spin-2, which Weinberg (1965) and others successfully proved to be the case.

The mass matrix shows the same pattern again, more or less (Fig. 7b). However, the Charge Matrix is a little different (Fig. 7a). There is a regression line splitting the data into two sections; 21 and 15 datapoints in size, respectively. Points (1, 4) and (4, 1) no longer belong to the pentonion side and (3, 3) no longer fits into the trionion matrix.

This same regression line can be also applied to the Mass Matrix (Fig. 7b). These are interesting results and they call to mind the Pythagorean understanding that all square numbers can be generated by the sum of two neighboring triangular numbers. In this case, we have 15 + 21 = 36.

36 is itself a triangular number, which suggests that the set of particles we have is one half of another set, which can either be 64 or 81 particles in size. We can, therefore, rearrange the masses into Pythagorean form in a (8×8) matrix (Fig. 8). The result shows a distinct asymmetry along the anti-diagonal. It looks like a group of Higgs Bosons are hovering around the top quark. Since the Higgs gives mass to the elementary particles, perhaps this grouping is why the top quark is so heavy. In fact, according to the Standard Model, this 'mobbing' of the *t*quark is precisely the reason for its unusually high mass.

0	0.0047	50	1.28	91.19	173.1	124.97	0
0.0022	0	2.2e-08	0.10566	124.97	124.97	124.97	--
50	0.000511	0	0.0182	0	124.97	--	--
0.96	0.00017	1.7768	0	0	--	--	--
91.19	124.97	0	0	--	--	--	--
4.18	0	0	--	--	--	--	--
0	124.97	--	--	--	--	--	--
124.97	--	--	--	--	--	--	--

Fig. 8. The order 8 mass matrix using triangular numbers.

The true asymmetry lies in the GeV triplets; 124.97, 173.1, 124.97 {at coordinates (0,4), (0,5), (1,4)} and 0, 4.18, 0 {at (5,0), (5,1), and (6,0)}. However, we can still see a sort of symmetry here in terms of respective magnitudes. The cells marked '- -' are as yet unknown quantities. But we can fill them in by simply mirroring the particles we already have into that region. This can lead to the new kind of symmetric Standard Model shown in Figure 9.

g	d	1W1Z	c	WZ	t	H_3	G_8
u	y	ve	μ	H^1	H^2	H^4	H^3
1W1Z	e	y	$v\tau$	G_1	H^4	H^2	t
s	$v\mu$	τ	G_4	G_5	G_1	H^1	WZ
WZ	H^1	y	G_3	G_4	$v\tau$	μ	c
b	G_6	G_7	y	τ	y	ve	1W1Z
G_2	H^2	G_6	H^1	$v\mu$	e	y	d
H^3	G_2	b	WZ	s	1W1Z	u	g

Fig. 9. The new kind of Super-Symmetry.

Super-Symmetric Octonions

As we might expect, this model is symmetric, along one axis. Along the other we see asymmetries, which once again show an imbalance of mass hovering around the top quark, and less around the bottom quark. Once again, this sheds light on the gross imbalance of mass apparent in the different quark flavors.

g	d	1W1Z	c	WZ	t	H^3	G_8
u	v	ve	μ	H^1	H^2	H^4	H^3
1W1Z	e	v	$v\tau$	G_1	H^4	H^2	t
s	$v\mu$	τ	G_4	G_5	G_1	H^1	WZ
WZ	H^1	v	G_3	G_4	$v\tau$	μ	c
b	G_6	G_7	v	τ	v	ve	1W1Z
G_2	H^2	G_6	H^1	$v\mu$	e	v	d
H^3	G_2	b	WZ	s	1W1Z	u	g

(a)

e7	e6	e5	e4	e3	e2	e1	1
e6	-e7	-e4	e5	-e2	e3	-1	e1
-e5	-e4	e7	e6	e1	-1	-e3	e2
-e4	e5	-e6	e7	-1	-e1	e2	e3
e3	e2	e1	-1	-e7	-e6	-e5	e4
e2	-e3	-1	-e1	e6	-e7	e4	e5
-e1	-1	e3	-e2	-e5	e4	e7	e6
-1	e1	-e2	-e3	e4	e5	-e6	e7

(b)

Fig. 10. (a) The Standard Model and (b) the flipped Octonions.

In the conventional Super-Symmetric model, the duplicate particles have heavier masses. In our version, the duplicate particles merely represent the anti-particles (or perhaps helicity). The anti-diagonal in Figure 9 consists of particles and anti-particles. However, since it is all composed of gauge bosons anyway, this isn't a problem. The same is true of gauge bosons that exist off this anti-trace line.

Now that we have a (8×8) matrix, we can easily associate these with the traditional octonions (Fig. 10). This is not a concrete association, however, and the author has included it merely for the purpose of being completely thorough in our investigation. In order for these two systems to match, one of them needs to be flipped along its *y*-axis.

g	*d*	1*W*1*Z*	*c*	*WZ*	*t*	H^3	G_8	--
u	*y*	*ve*	*μ*	H^1	H^2	H^4	H^3	--
1*W*1*Z*	*e*	*y*	*vτ*	G_1	H^4	H^2	*t*	--
s	*vμ*	*τ*	G_4	G_5	G_1	H^1	*WZ*	--
WZ	H^1	*y*	G_3	G_4	*vτ*	*μ*	*c*	--
b	G_6	G_7	*y*	*τ*	*y*	*ve*	1*W*1*Z*	--
G_2	H^2	G_6	H^1	*vμ*	*e*	*y*	*d*	--
H^3	G_2	*b*	*WZ*	*s*	1*W*1*Z*	*u*	g	--
--	--	--	--	--	--	--	--	--

(a)

(b)

$e_0 \mapsto 1$	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
e_1	-1	e_3	$-e_2$	e_5	$-e_4$	$-e_7$	e_6	e_9	$-e_8$	$-e_{11}$	e_{10}	$-e_{13}$	e_{12}	e_{15}	$-e_{14}$
e_2	$-e_3$	-1	e_1	e_6	e_7	$-e_4$	$-e_5$	e_{10}	e_{11}	$-e_8$	$-e_9$	$-e_{14}$	$-e_{15}$	e_{12}	e_{13}
e_3	e_2	$-e_1$	-1	e_7	$-e_6$	e_5	$-e_4$	e_{11}	$-e_{10}$	e_9	$-e_8$	$-e_{15}$	e_{14}	$-e_{13}$	e_{12}
e_4	$-e_5$	$-e_6$	$-e_7$	-1	e_1	e_2	e_3	e_{12}	e_{13}	e_{14}	e_{15}	$-e_8$	$-e_9$	$-e_{10}$	$-e_{11}$
e_5	e_4	$-e_7$	e_6	$-e_1$	-1	$-e_3$	e_2	e_{13}	$-e_{12}$	e_{15}	$-e_{14}$	e_9	$-e_8$	e_{11}	$-e_{10}$
e_6	e_7	e_4	$-e_5$	$-e_2$	e_3	-1	$-e_1$	e_{14}	$-e_{15}$	$-e_{12}$	e_{13}	e_{10}	$-e_{11}$	$-e_8$	e_9
e_7	$-e_6$	e_5	e_4	$-e_3$	$-e_2$	e_1	-1	e_{15}	e_{14}	$-e_{13}$	$-e_{12}$	e_{11}	e_{10}	$-e_9$	$-e_8$
e_8	$-e_9$	$-e_{10}$	$-e_{11}$	$-e_{12}$	$-e_{13}$	$-e_{14}$	$-e_{15}$	-1	e_1	e_2	e_3	e_4	e_5	e_6	e_7
e_9	e_8	$-e_{11}$	e_{10}	$-e_{13}$	e_{12}	e_{15}	$-e_{14}$	$-e_1$	-1	$-e_3$	e_2	$-e_5$	e_4	e_7	$-e_6$
e_{10}	e_{11}	e_8	$-e_9$	$-e_{14}$	$-e_{15}$	e_{12}	e_{13}	$-e_2$	e_3	-1	$-e_1$	$-e_6$	$-e_7$	e_4	e_5
e_{11}	$-e_{10}$	e_9	e_8	$-e_{15}$	e_{14}	$-e_{13}$	e_{12}	$-e_3$	$-e_2$	e_1	-1	$-e_7$	e_6	$-e_5$	e_4
e_{12}	e_{13}	e_{14}	e_{15}	e_8	$-e_9$	$-e_{10}$	$-e_{11}$	$-e_4$	e_5	e_6	e_7	-1	$-e_1$	$-e_2$	$-e_3$
e_{13}	$-e_{12}$	e_{15}	$-e_{14}$	e_9	e_8	e_{11}	$-e_{10}$	$-e_5$	$-e_4$	e_7	$-e_6$	e_1	-1	e_3	$-e_2$
e_{14}	$-e_{15}$	$-e_{12}$	e_{13}	e_{10}	$-e_{11}$	e_8	e_9	$-e_6$	$-e_7$	$-e_4$	e_5	e_2	$-e_3$	-1	e_1
e_{15}	e_{14}	$-e_{13}$	$-e_{12}$	e_{11}	e_{10}	$-e_9$	e_8	$-e_7$	e_6	$-e_5$	$-e_4$	e_3	e_2	$-e_1$	-1

Fig. 11. (a) The expanded Nononion matrix and (b) the Sedenions.

There are noticeable issues with the comparison. For starters, the top quark is related to e_2 but it has no corresponding $-e_2$ value to represent its anti-particle. The same is true of the charm and down quark. It is possible that one of the other 480 possible octonion multiplication tables over the Reals would produce better results but even if they did little of benefit can emerge from it.

One reason for this is that we are only using the (8×8) matrix. It obviously makes more sense to use the (9×9) matrix (Fig. 11a). This would be a Nononion matrix (it is pronounced exactly how it is written; nononion) and would thereby bring Sedenions into the mix (Fig. 11b). However, it would also lead us to a situation whereby many of the particles and their properties are completely unknown. As for what these new particles might be, we can speculate that they are of the 'Dark Matter' variety. This method may give us a means of inferring the properties of some of these particles based on the particles' properties in the known Standard Model.

The Dark Matter is generally considered to be nonbaryonic, as it does not interact with the main forces. Here the author will advocate for a kind of dark matter which has as much variation and ability to interact with itself as does ordinary matter.

The Sedenions

Now, we are definitely getting somewhere. Here we have four copies of our reflected Standard Model, each corresponding to a Sedenion multiplication (Fig. 12). We've solved the particle anti-particle issue and there's no need to flip the Sedenions. The top left-hand quadrant is the original matrix corresponding to the Octonions, the bottom right hand matrix is nearly the same. It differs in so much as all of the signs are inverted; allowing for the anti-particles (opposite signs in the same quadrant likely indicate helicity). However, it is also made by the multiplication of purely Sedenionic numbers with themselves.

The two other quadrants represent the Dark Matter particles and anti-particles. They are the result of left and right multiplication of pure Octonions with Sedenions. Therefore, we can say (if the analogy holds) that the dark matter is the result of Sedenion multiplication between matter and anti-matter. This means that it exists at 'right angles' to ordinary matter and may explain why it isn't seen to interact. Furthermore, anti-matter is actually the result of the dark matter multiplied by the dark anti-matter.

Now that we have this, let's see what we can do with it. We'll start with the tau (τ). The matter tau is: $\tau = (e_3.e_2)$, the anti-matter tau is $\tau^* = (e_{11}.e_{10})$, the dark matter tau is $|\tau| = (e_{11}.e_2)$ or $(e_3.e_{10})$, the dark matter anti-tau is $|\tau^*| = (e_5.e_{12})$ or $(e_{13}.e_4)$. Now, let's look at what these numbers breakdown to:

$$e_4 = WZ,\ e_2 = 1W,\ e_{11} = s,\ e_5 = b,$$
$$e_{13} = b,\ e_{10} = 1W,\ e_3 = c$$

So, what can be made from all of these? That's right. We can use these particles to make bottom quark decay diagrams shown in Figure 13, as well as the *b*-quark decay diagrams shown in Figure 14.

It is further interesting (although no doubt expected) that this reaction should begin with the tau and the anti-tau and then transition to the *W* boson and bottom quarks, etc. There are probably many more such interactions lurking in the Sedenions. However, how many are not right? Presumably, there are some. And where are the infinite number of interactions, we would expect from quantum physics? At a glance, they're not there.

While the Sedenions may not provide all of the answers, as far as the author is concerned, this has been a satisfying investigation into the subject. Compared with other methods of fitting the subatomic particles into a kind of ordered symmetry using complex numbers, it is this one

	e_0	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}
e_0	g	d	1W	c	WZ	t	H^3	G_8	g	d	1W	c	WZ	t	H^3	G_8
e_1	u	y	ve	μ	H^1	H^2	H^4	H^3	u	y	ve	μ	H^1	H^2	H^4	H^3
e_2	1W	e	y	vτ	G_1	H^4	H^2	t	1W	e	y	vτ	G_1	H^4	H^2	t
e_3	s	vμ	τ	G_4	G_5	G_1	H^1	WZ	s	vμ	τ	G_4	G_5	G_1	H^1	WZ
e_4	WZ	H^1	y	G_3	G_4	vτ	μ	c	WZ	H^1	y	G_3	G_4	vτ	μ	c
e_5	b	G_6	G_7	y	τ	y	ve	1W	b	G_6	G_7	y	τ	y	ve	1W
e_6	G_2	H^2	G_6	H^1	vμ	e	y	d	G_2	H^2	G_6	H^1	vμ	e	y	d
e_7	H^3	G_2	b	WZ	s	1W	u	g	H^3	G_2	b	WZ	s	1W	u	g
e_8	g	d	1W	c	WZ	t	H^3	G_8	g	d	1W	c	WZ	t	H^3	G_8
e_9	u	y	ve	μ	H^1	H^2	H^4	H^3	u	y	ve	μ	H^1	H^2	H^4	H^3
e_{10}	1W	e	y	vτ	G_1	H^4	H^2	t	1W	e	y	vτ	G_1	H^4	H^2	t
e_{11}	s	vμ	τ	G_4	G_5	G_1	H^1	WZ	s	vμ	τ	G_4	G_5	G_1	H^1	WZ
e_{12}	WZ	H^1	y	G_3	G_4	vτ	μ	c	WZ	H^1	y	G_3	G_4	vτ	μ	c
e_{13}	b	G_6	G_7	y	τ	y	ve	1W	b	G_6	G_7	y	τ	y	ve	1W
e_{14}	G_2	H^2	G_6	H^1	vμ	e	y	d	G_2	H^2	G_6	H^1	vμ	e	y	d
e_{15}	H^3	G_2	b	WZ	s	1W	u	g	H^3	G_2	b	WZ	s	1W	u	g

Fig. 12. The eight copies of the Standard Model corresponding to the Sedenions.

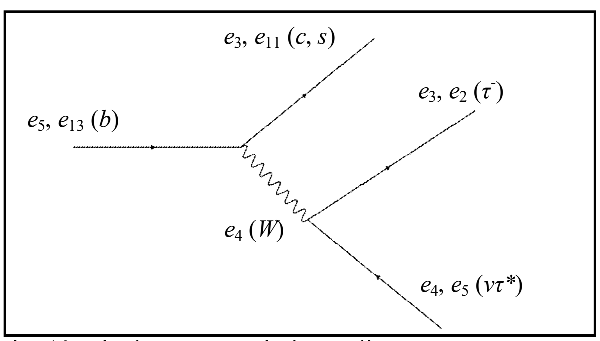

Fig. 13. The bottom quark decay diagram.

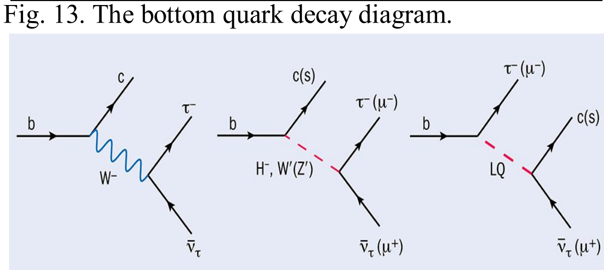

Fig. 14. More b-quark decay diagrams (https://cerncourier.com/a/the-flavour-of-new-physics/).

that provides the best results, in this author's opinion. Overall, this suggests the important relationship between these hyper complex numbers with that of the Standard Model and particle physics.

CONCLUSION

All 32 particles of the Standard Model (along with several new ones) can be represented in the form of a (6×6) Pentonionic grid. This configuration explains why there are only three flavours of matter in existence. Massless bosons are shown to conform with the multiplication of like-valued imaginary numbers, i.e. $i \times i = -1$. The Pentonion grid also reveals an important relationship between the graviton and the 8 color charges of the gluon and hints that the two linear dependent gluons might help with the unification of the fundamental forces. A deeper structure of the Pentonions is hinted at via triangular numbers. This structure is expanded into an (8×8) matrix, and finally a (16×16) Sedenion grid, leading to a new kind of symmetric structure of particles, antiparticles, as well as the dark matter particles and antiparticles and helicity.

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